Uncertainty, Default, and Risk

(Welch, Chapter 06)

Ivo Welch

UCLA Anderson School, Corporate Finance, Winter 2017

December 15, 2016

Did you bring your calculator? Did you read these notes and the chapter ahead of time?

Maintained Assumptions

In this part (consisting of three chapters), we maintain the assumptions of the previous chapter:

- We assume **perfect markets**, so we assume four market features:
  1. No differences in opinion.
  2. No taxes.
  3. No transaction costs.
  4. No big sellers/buyers—infinitely many clones that can buy or sell.
- We already allowed for unequal rates of returns in each period.
- **But we now allow uncertainty. So, we do *not* know in advance what the rates of return on every project are.**

We now need to predict (describe) the future. For this, we need statistics.

Warning: This Chapter May Be Illegal in Some States

Attend the rest of this course at your own risk:

> *Persons pretending to forecast the future shall be considered disorderly under subdivision 3, section 901 of the criminal code and liable to a fine of $ 250 and/or six months in prison.*
>
> ** (Section 889, New York State Code of Criminal Procedure.)*

Statistics and Random Variables

- Covered fully in your sadistics [sic] course.
- A **random variable** (denoted by a tilde over a variable in a more formal statistics course) is not an ordinary variable.
  - A random variable can take on a whole range of possibilities, which can be drawn in a histogram. In a sense, a random variable is more like a function of a randomizing device (e.g., a coin): it needs to obtain a value from this randomizing device.
  - For example, the random variable D may be:
    $+\$10$ if heads (prob 50%), $+\$50$ if tails (prob 50%).
  - It is often useful to think of a random variable as being the histogram itself. At least this is how I think of them.
- In most statistical applications, users usually *assume* that they know the histogram, but not the draw (outcome).
  - This is great for a coin, die, or roulette. It sucks for the stock market, where we do not understand the underlying physics.
  - On occasion, I will warn you about this **big** leap of faith.

Our main random variable example will be the payoff you get after a die is thrown:

$$\text{“}1\text{”} = -\$6; \text{“}2\text{”} = \$36; \text{“}3\text{”} = -\$12; \text{“}4 \;\text{ to }\; 6\text{”} = \$150.$$

Let's call this variable D. Draw the histogram

The Expected Value of a RV

- $E(\cdot)$ is the abbreviation. For example, $E(D)$.
- Think of the expected value (mean) of a random variable as the average if you repeat the experiment infinitely many times.
- From a RV's histogram, you can caculate the expected value by multiplying each outcome by its probability, and adding them up.

What is the expected payoff of our random variable D?

Is E(D) a random variable, too?

Is the expected payoff always the most likely outcome?

- Are half of all outcomes always below the mean?
- What is the mean number of testicles per human?
- What is the median?
- What is the mode?
- (What is the standard deviation?)

What is the expected value of the die-squared, $E(D^2)$?
Recall D

$$\text{“1”} = -\$6; \text{“2”} = \$36; \text{“3”} = -\$12; \text{“4 to 6”} = \$150.$$

Is $E[(D^2)]$ the same as $[E(D)]^2$?

PS: $2 \times E(D) + 1 = E(2 \times D + 1)$

What is a fair bet? What would it take for the above die-bet example to become a fair bet?

Variance

- The variance is $\text{Var}(D) = E\{[D - E(D)]^2\}$: roughly speaking, the **expected squared deviation from the mean**. This is a pseudo-intuitive name.
- Think of the true unknown variance of a random variable as the computed squared deviation from the mean if you draw infinitely many realizations.
  - In your statistics class, you may also have encountered the "variance of the mean" estimate. This is a related concept used to test how likely it is that the underlying mean is actually zero. However, it is *not* the variance of a random variable.
- From the RV's histogram, to obtain the variance, multiply each squared deviation from the mean by its probability, and add 'em up. Example:

State	Prob	Outcome	Outcome –Mean	Squared
"1"	1/6	–\$6	–\$84	\$\$7,056
"2"	1/6	\$36	–\$42	\$\$1,764
"3"	1/6	–\$12	–\$90	\$\$8,100
"4"-"6"	3/6	\$150	+\$72	\$\$5,184
Weighted Mean:		\$78	\$0	\$\$5,412

- The units on a variance are usually incomprehensible.

Standard Deviation

- The standard deviation is the square-root of the variance:

$$\text{SD} = \sqrt{\$\$5,412} \approx \$73.57$$

- Intuitively, think of the standard-deviation is the "typical deviation from the mean" of the next draw.
  - This is not entirely correct, but it is close enough and often helps the intuition.
- If a variance is higher, then the standard deviation is higher.
- When people talk about risk, they often use phrasing such as "if the variance is higher." They could equally well say "if the standard deviation is higher." In fact, they often mean the latter.
  - This is economics in action. People are mouth-lazy. Variance has 2 syllables, standard deviation has 5 syllables.

Big Leap of Faith — What is the True Histogram?

- Assuming that we know the histogram is good for a throw of a die, where we know the physics.
- We do not really know the histogram for the rate of return on the stock market.
- Therefore, we pretend that the many historical outcome realizations of rates of returns can proxy for the true unknown histogram of rates of returns, and then we pretend that this histogram applies to future rates of return, too.
  - This translation of the historical outcome histogram (distribution) into the future outcome histogram (distribution) is a heroic assumption—but it is often the only reasonable information that you have. You often have no better alternatives.
  - If we use the historical realized distribution, some statistics require a small correction. (This is the least of our problems.) I will tell you soon.
  - Extrapolation works reasonably well for (short-term) standard deviations, but almost always poorly for means. Just because Google had an average 50% rate of return per year in the past does not mean it will have an expected rate of return of 50% per year in the future.
  - For example, what is the expected rate of return on the stock market? Is it the same that it was historically?
  - You must be mindful of where histo-extrapolation works and where it fails.
- If you assume you know the histogram, then you can calculate the expectation.

What is the expected mean and standard deviation if stock returns followed this historical distribution:

$$+10\% \quad -5\% \quad +20\% \quad +15\%$$

AGAIN Means and Variance Estimates For Stock Returns

- We estimate a distribution of future rates of return from historical rate of return realizations.
- We assume each historical realization was an equally likely draw.
- When we compute the variance from a histogram that relies on a sample (not the known histogram of the population), we adjust by dividing not by N, but $N-1$.
  - *Why Excel and Stats packages sometimes give different answers for Var and Sdv:* We divide by N, because we presume you know the population. Excel and stats packages without the 'p' at the tail end of the function name divide by $N-1$, because they presume you know only a sample drawn from the population. Usually, this makes little difference if you have reasonably large datasets—but it makes a difference in our small $N = 4$ datasets. So, you must use stdevp and varp, and not stdev and var to get the same results as those we compute in these slides in class.
- Let me warn you again: Do not trust the historical means blindly especially when it comes to predicting future expected rates of returns. For individual stocks (rather than big diversified portfolios), this would be incredibly bad. Even for big diversified portfolios, this is a big leap of faith.
- *In contrast, (recent) historical variances (and covariances and standard deviations) are usually better predictors of (short-term) future variances (and covariances and standard deviations).*

  Advice: use a couple of years, say 1–5, of historical data to estimate them.
- However, a short time-series of historical numbers is usually not reliable enough to calculate tail-risk—the probability of a complete blow-up. How do you estimate the risk of the next Space-X rocket exploding?

Preferences

If I offer you a bet of $+\$1$ if heads and –\$1 if tails, you pick a coin and someone else in class to throw it [at least 5 yards], would you be willing to take this bet? If not, how much would I have to pay you?

When is risk neutrality or low risk-aversion a good assumption?

Why do people climb mountains, drive motorcyles, play the lottery?

Default ("Credit") Risk

Most loans have credit risk, in that the borrower can default.

Loosely, it is the part of the return that is promised but will be lost on average because the borrower goes belly-up.

Can the U.S. government default? Do Treasury securities have any default (credit) risk?

Working Example

Henceforth, assume that a government bond costing \$200 promises a 5% interest rate, i.e., \$210.

- Assume you are risk-neutral.
- I want to borrow \$200 from you. I promise to repay \$210.
- However, I may go bankrupt in 1 out of 100 cases, in which case I can repay only \$50.

What is your promised rate of return on my personal bond?

Do I promise to give you the same rate of return as the Treasury?

What would you expect my personal bond to return?

If you extend this loan to me, what rate of return would you expect my bond to give you?

Would you prefer to make this loan or to put your money into the 5% government bond?

How much money would you give me in exchange for my promise to pay you \$210?

If the WSJ were to print my bond's interest rate, what interest rate would it print?

Are (WSJ) quoted interest rates on risky bonds expected rates?

In the real world, would this interest rate really be high enough?

Probably not. There would also be risk premia (for investors to be willing to take on risk); and imperfect-markets premia, such as liquidity premia (because it is harder to resell untraded small-fry risk). These and other premia will be discussed in a later chapters.

Default (Credit) and Risk Premia

- The default premium is compensation to make you break even. It is required to get you to participate even if you are risk-neutral.
  - If you repeat the investment infinitely many times, the average default payment is 0. You get a positive amount if everything goes well, and a negative amount if default occurs. That is, you come out even.
- The risk premium is extra compensation that gets you above the time premium, and it is only required to get you to participate if you are risk-averse.
  - If you repeat the investment infinitely many times, the risk premium will allow you to earn more than an investor holding Treasuries will earn.

Never confuse the credit premium and the risk premium.

In our world, with a Treasury rate of 5.00% and a quoted bond of 5.81%, the risk premium was still zero.

- **Warning:** You must be clear about the distinction between default premia and risk premia. Make sure you know what they are about, and know the difference between these concepts!

Important Generalization of Premium Decomposition

In a risk-neutral world:

Quoted (=Promised) Interest Rate $\geq$ Expected Interest Rate.

Quoted Interest Rate = Time Premium + Default Premium

Expected Interest Rate = Time Premium

- We are ignoring other premia for now, such as risk premia, liquidity premia, tax premia, contract premia, etc.

In our example, the promised interest rate was 5.81%, the time premium is 5.00%, the default premium was 0.81%, and the risk premium is 0.00%.

- Risk premia for "fairly safe" bonds from "large, safe" companies are not too high; it is the default premia that jacks up the quoted interest rates.
- IRR computed from promised cash flows is a promised IRR. It is what the WSJ prints.
- The promised IRR is never used in the IRR Capital Budgeting Rule. For capital budgeting purposes, you must use an IRR computed from expected cash flows, not from promised cash flows.

Default (Credit) in Net Present Values

In PV applications, you have to use

$$\frac{\text{E}(\textbf{Cash Flow})}{[1+\text{E}(\textbf{Discount Rate})]}$$

In the real world, users often mess up PV calculations because they do not understand the distinction between default rates and risk rates. You must use **expected** values in both the numerator and the denominator.

- The expected payoff is in the numerator, and it takes care of the default risk of our project. The correct PV of our loan promising \$210

  $$\text{PV} = \frac{\text{E(Cash Flow)}}{[1+\text{E(Discount Rate)}]} = \frac{\$208.40}{(1+5\%)} \approx \$198.47$$

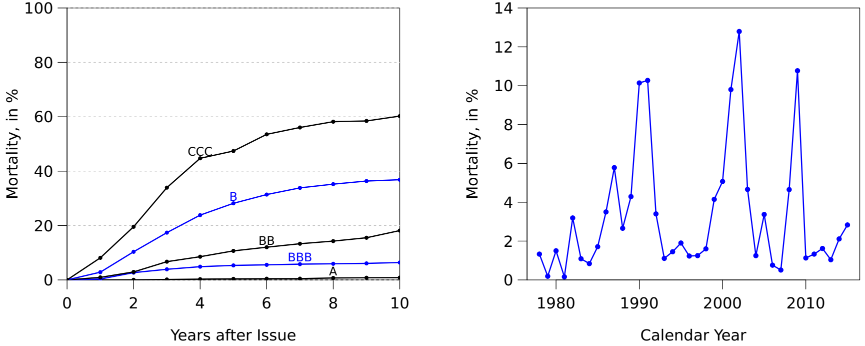
  It is **not** the promised amount of \$210 divided by the cost of capital
- The expected discount rate—*not the promised rate of return*—is in the denominator. It is the opportunity cost of capital on other projects, quoted in terms of their expected rates of return, not in terms of their promised rates of return.
  You can't use the **promised** rate of return of opportunities elsewhere as your coc.
- In a risk-neutral world, every project has the same denominating cost of capital (here 5%), regardless of how likely the project or bond is to pay what it promises.

Credit Ratings

Large corporations have credit ratings, too, ranging from AAA (best) to F.

- Typical AAA firm has a $\sim 0\%$ probability of default over 10 years.
- Typical B firm has a 20% probability of one non-payment over 5 years.
- Typical C firm has a 50% probability of one non-payment over 6–8 years.

U.S. History



100
80
60
40
20
0
Mortality, in %
CCC
B
BB
BBB
A
0
2
4
6
8
10
Years after Issue
14
12
10
8
6
4
2
0
Mortality, in %
1980
1990
2000
2010
Calendar Year

More Default Risk or Risk Premium?

Most of the yield spread of corporate bonds is due to the chance of default (i.e., the credit spread).

For example, if a Boston Celtics = 9.4%, whereas a similar Treasury = 5.6%, then I would guesstimate that the Celtics bond pays off, on average, say, about 6.0%. 3.4% is then the default risk, and 0.4% is the risk premium (including possibly the liquidity premium.)

The Single-Most Important Lesson under Uncertainty

Never ever confuse expected rates with (higher) promised rates.

- The 9.4% from the Boston Celtics is not expected!
- Newspapers and websites virtually never report expected rates.
- If you use a promised or quoted cash flow where you have to use an expected cash flow (i.e., you mix up these two), do not mention **under any circumstances** that you took this finance course with me as your instructor.

Credit (Default) Swaps (CDS)

- Nowadays, you can buy insurance against default, called credit (default) swaps. The financial crisis of 2008 has made them famous. They played a central role.
- This market is over-the-counter (OTC). Sellers are often hedge funds who want to speculate on default. Buyers are often mutual funds or pension funds who want to reduce their risk exposure.
- In the event of default, the seller of CDS may either have to pay the CDS buyer a fixed amount, or allow the CDS buyer to sell the bond for a pre-agreed price to the CDS seller upfront, as negotiated up front.
  - However, default events—when the buyer of the CDS may demand payment—are declared by a committee of conflicted I-bankers. Huh?
- In 2016, there was more than $17 trillion of single-name credit swaps outstanding.
- This is a rather opaque market—it is possible that the risk of credit is no longer with the holders of the corporate debt.
  - Risk is somewhat similar to the housing derivative risk—an obscure bank in Germany may blow up over housing trouble in Kansas.
  - A fund can buy the bonds, insure itself many times over against default with a CDS, and then vote to try to drive the firm itself into bankruptcy.
- However, as a buyer of a CDS, you will also have to worry about whether the issuer of the CDS will go bankrupt itself.

Uncertainty in CapBudg: PV With Debt and Equity

We have already covered rates of return and NPV under uncertain future cash flows. This book chapter represents another important conceptual leap:

- Introduces Payoff Tables and Contingent Claims Valuation.
- Introduces Bonds vs. Levered Equity.
- Introduces Bond Risk vs. Equity Risk.

These are not minor topics, but some of the most important concepts in finance. They are big deals!

- Every investment opportunity in our perfect world is fairly priced.
- You can see yourself as one of two types of investors:

  A lender
  : who has provided capital in exchange for the promise of a fixed amount of money. [called leverage]

  A levered homeowner
  : (often just called homeowner), who owns the house only with the bundled obligation to repay the loan.

A Financed Project (House/Firm/...)

Next Year's Payoffs	Probability	
\$100	90%	(Sunshine)
\$50	10%	(Hurricane)

- The expected rate of return on 1-year Treasuries (and all other 1-year financial instruments) is 5%.
- The world is risk-neutral.

This is the project example for all the following pages.

What is the appropriate price for this project?

What is the rate of return on the project in the good state (=promised rate of return)?

What is the rate of return on the project in the bad state?

What is the expected rate of return on the project?

Levered Equity (Stock) + Risk-free Bond

You can finance the project in one of two ways:

- You can buy it outright (with \$0 mortgage) with financing from your life's savings account.
- You can buy it with a mortgage and a smaller sum from your life's savings account. You then own just the residual equity called **Levered Equity** or **Levered Stock** or just **Stock**. It is what you get to keep after you will have repaid the debt.
- Let's work with a specific example. Let's finance your purchase with a loan (=bond) promising \$50.

We assume that financial markets are still perfect, as before.

Scheme 2

- In the good state, how much do bond and levered equity receive?
- In the bad state, how much do bond and levered equity receive?

What are the appropriate prices for the bond and the levered equity? [PTO]

The Main Diagram

Project Payoffs	Scheme 1: Firm, FM (=100% Equity)	Scheme 2: Bond, DT (Promise=$50)	Scheme 2: Levered Equity, EQ
prob(G)=	r=______	r=______	r=______
prob(B)=	r=______	r=______	r=______
E(Payoff) (= E(C))			
E(Rate of Return) (= E(r))			
Discounted Price P_0			
% Financing			

Histogram Prep

- In the good state, what is the *rate of return* that the bond and the levered equity receive?
- In the bad state, what is the *rate of return* that the bond and the levered equity receive?

Draw a histogram of the return distributions for all three forms of ownership considered sofar.

Is full project ownership (=zero leverage) or levered project ownership riskier?

Is full project ownership (=zero leverage) or bond ownership riskier?

Limited Liability

- Limited Liability : you are on the hook only for what you invested, and no more.
- Limited Liability was a central innovation in finance in the Renaissance. (It was not known in Medieval or Roman times.) It came into wide use in the 18th and 19th century.
- It made it possible for owners to hand control to specialists and not worry for their entire holdings.
- The President of Columbia University wrote in 1911 that its discovery was more important than that of steam and electricity.

Bond Promising $70 Next Year

Usually, equity has **limited liability**, which is how we will use it henceforth in the remainder of the course.

- Now price a bond with a promise of $70.
- Enter everything you know.
- Work down the project without financing.
- Work down the pricing of the bond.
- Work back up the pricing of the equity.
- Use the next page

(PS: This works exactly the same way with more than two possible outcomes.)

Bond Promising \$70

	Scheme 1	Scheme 2	
Project Payoffs	Firm, FM (=100% Equity)	Bond, DT (Promise=\$70)	Levered Equity, EQ
prob(G)=	r=______	r=______	r=______
prob(B)=	r=______	r=______	r=______
E(Payoff) (= E(C))			
E(Rate of Return) (= E(r))			
Discounted Price P_0			
% Financing			

How much do you need to promise to borrow \$70 today?

Project Payoffs	Scheme 1: Firm, FM (=100% Equity)	Scheme 2: Bond, DT (Promise=??)	Scheme 2: Levered Equity, EQ
prob(G)=	r=______	r=______	r=______
prob(B)=	r=______	r=______	r=______
E(Payoff) (= E(C))			
E(Rate of Return) (= E(r))			
Discounted Price P_0			
% Financing			

What happens to the riskiness of the **stock** when more mortgage (say, \$70 rather than \$1) is taken on?

What happens to the riskiness of the **mortgage** when more mortgage (say, \$70 rather than \$1) is taken on?

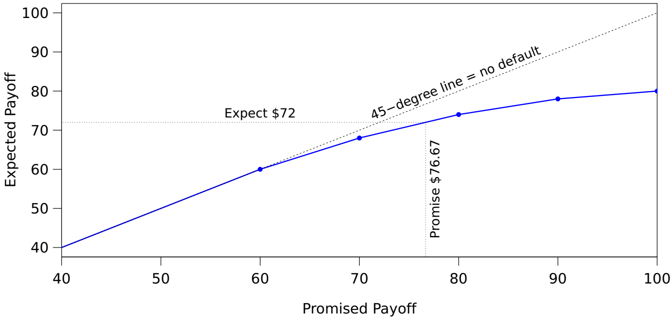
What happens to the riskiness of the “firm” (the house overall) when more mortgage is taken on?

A Broader View of Leverage

- Leverage = Small movement in lever can create much bigger or smaller movement elsewhere (in the equity).
- Leverage = The safer part is "outsourced." Small movement in underlying project can make levered ownership much riskier — more upside and more downside.
- Can be done in various ways:
  - With Financial Leverage, as in the example above.
  - With Operational Leverage. Example: Instead of owning safe building and risky technology (together = project medium risky), just lease the safe building. All your money is now in risky technology.

More than Two Possible Outcomes

Everything you learned generalizes.



In fact, everything can be done with normally distributed returns, too.
In this case, the curve would be smooth.

Recall that you can discount nominal payouts with nominal expected rates of return and come to the same result as with real payouts with real expected rates of return.

Can you discount promised payouts with promised rates of return and come to the same result as when you discount expected payouts with expected rates of return?