Investing Choices and Risk Measures

(Welch, Chapter 08)

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Did you bring your calculator? Did you read these notes and the chapter ahead of time?

Maintained Assumptions

In this part (starting from the previous chapter), we maintain the same assumptions:

- We assume **perfect markets**, so we assume four market features:
  1. No differences in opinion.
  2. No taxes.
  3. No transaction costs.
  4. No big sellers/buyers—we have infinitely many clones that can buy or sell.
- We already allow for unequal rates of returns in each period.
- We already allow for uncertainty. So, we do *not* know in advance what the rates of return on every project are.

- **But in contrast to Chapters 6, we no longer assume risk-neutrality. We are allowing for risk aversion now.**
  - Recall Chapter 4, in which we found out that you are risk-averse.

- In this chapter, we lay the groundwork for understanding how investors choose among many different projects.
- You need this [a] to think as a manager about your company's investment risk; [b] more importantly to think about your "opportunity cost of capital," E(r).

OMIT: What Preference Assumptions Buy Us

What do we need to compare stocks if we can only invest in one?

	State 1	State 2	State 3	
Base Stock A	–5%	5%	15%	
Stock B1	–6%	4%	10%	same, but 1% less in all states
Stock B2	–6%	6%	5%	better in state 2, worse in 1 and 3
Stock B3	–10%	5%	20%	same mean, higher variance
Stock B4	–10%	6%	20%	higher mean, higher variance

- Everyone prefers A to B1.
- Not everyone prefers A to B2—you must assume that one cares about states equally to prefer A. (Simplest example: [a] you are only alive in state 2; or [b] the probability of state 2 is 99%.) (Nerd note: in asset pricing models, we use marginal utilities (pricing kernels) to handle [a]. Essentially, when we care more about a state, it is like assuming there is a higher probability of this state occurring.)
- Risk-averse investors would prefer A to B3.
- For B4, we need to specify how we want to trade off risk vs. return.

We will not talk more about (state) preferences, but just assume that you and our other investors care about mean (reward) and standard deviation (risk). They like more of the former and less of the latter.

Some Particular Investments

Scenarios		Assets A	B	C	D
1/4	Yellow	−4	−1	−1.25	+3
1/4	Red	−4	+9	+1.25	+13
1/4	Green	+6	+9	+3.75	+3
1/4	Blue	+6	−1	+1.25	−7

We will be using these four assets (A to D) in these slides. You can think of these as rates of return (or, if you prefer, as dollar returns instead of rates of return).

What are the rewards of your above investment opportunities?

What are the risks of your above investment opportunities?

Population vs. Sample Statistics

- If these returns are just representative historical realizations from a population, you would divide by 3, not 4 in your computation of the variance.
- In real life, we never have population statistics. We only have historical sample statistics. And we really should not trust the historical statistics either—but we do because this is the best alternative.

The standard deviation as a meaningful measure of risk applies only to your overall portfolio. You do not care about the standard deviation of your individual securities.

Complete the table

	A	B	C	D	$A-\overline{A}$	$B-\overline{B}$	$C-\overline{C}$	$D-\overline{D}$
Yellow	-4	-1	-1.25	$+3$	-5	-5	-2.5	0
Red	-4	$+9$	$+1.25$	$+13$	-5	$+5$	0.0	10
Green	$+6$	$+9$	$+3.75$	$+3$	$+5$	$+5$	2.5	0
Blue	$+6$	-1	$+1.25$	-7	$+5$	-5	0.0	10
Mean (E)	1	4	1.25	3				
Var								
Sdv								

What's the risk of an (equal-weighted) pfio of A and B?

(HINT: First compute the rates of returns of the combination portfolio in each state.)

Is the average portfolio or are the individual components riskier? Why?

What kind of portfolio would you—a smart investor—hold?

In real life, what portfolios should smart investors with risk-aversion hold?

What is your portfolio risk if you add C to your portfolio vs. if you add D to your portfolio?

	A	B	C	D	Half A, half **C**	Half A, half **D**
Yellow	-4	-1	-1.25	$+3$		
Red	-4	$+9$	$+1.25$	$+13$		
Green	$+6$	$+9$	$+3.75$	$+3$		
Blue	$+6$	-1	$+1.25$	-7		
Mean (E)	1	4	1.25	3		
Var	25	25	3.125	50		
Sdv	5	5	1.77	7.07		

Is C or D the riskier investment in itself?

If you already own A, is C or D the riskier addition?

Why?

If investors are smart, what is their A?

If you are selling to smart investors either C or D, for which of these two projects do you think will investors clamor to invest in your project (i.e., accept a lower expected rate of return)?

The fundamental insight of investments

- Investors (should) care about overall portfolio risk, not about the constituent component risk.
- From a corporate managerial perspective, it is not your projects that are low risk in themselves that are highly desirable for your investors, but projects which wiggle opposite to the rest of their portfolios

What is the synchronicity (correlation or beta) of our project that may matter to what our investors like?

Reminder

Note: M=A.

	M	B	C	D	$M-\overline{M}$	$B-\overline{B}$	$C-\overline{C}$	$D-\overline{D}$
Yellow	−4	−1	−1.25	+3	−5	−5	−2.5	0
Red	−4	+9	+1.25	+13	−5	+5	0.0	10
Green	+6	+9	+3.75	+3	+5	+5	2.5	0
Blue	+6	−1	+1.25	−7	+5	−5	0.0	10
Mean (E)	1	4	1.25	3				
Var	25	25	3.125	50	25	25	3.125	50
Sdv	5	5	1.77	7.07	5	5	1.77	7.07

How do you compute covariance, correlation, and beta?

- The covariance is the average sum of the cross-products:

$$(A-\overline{A})\times(B-\overline{B}) = 25, -25, 25, -25.\ \text{cov} = 0$$

$$(A-\overline{A})\times(C-\overline{C}) = 12.5, 0, 12.5, 0.\ \text{cov} = 6.25$$

$$(A-\overline{A})\times(D-\overline{D}) = 0, -50, 0, -50.\ \text{cov} = (-25)$$

  (Intuitively, why are we demeaning and multiplying? (use a graph))
- The beta (with respect to the market) is the covariance divided by the variance (of the market).

$$\beta_{C,A} = 6.25/25 = 0.25,\ \ C = 1 + 0.25\cdot A, \qquad (A = -1.5 + 2\cdot C)$$

$$\beta_{D,A} = -25/25 = -1,\ \ D = 4 - 1\cdot A, \qquad (A = 2.5 - 0.5\cdot D)$$

- The correlation is the covariance divided by the standard deviations of the two ingredients.

$$\text{cor}(A,C) \approx 0.7071 \qquad \text{cor}(A,D) \approx -0.7071$$

- The order matters for beta, but not for the covariance or correlation

Which is the best measure of risk?

- Covariance has uninterpretable units. Yuck.
- Correlation has a scale problem.
  - The correlation would tell us that a security with rates of return $R = (0.9, 1.0, 1.1, 1.0)$ has the same 70.7% correlation with A as $S = 1000 \cdot R - 1000 = (-100, 0, +100, 0)$ does—but \$100 of R will clearly contribute less risk to our portfolio M than \$100 of S.
- Therefore, we prefer measuring risk contribution of B or C by its market-beta with respect to A (here = M).
  - Beta is similar to correlation. It always has the same sign.
  - Beta can be interpreted as a slope. Put A (M) on the X axis, and your project B (or C) on the Y axis. A slope of 1 is a diagonal line. A slope of 0 is a horizontal line. A slope of ∞ is a vertical line.
  - Without alpha, beta tells you how an x% higher rate of return (than normal) in the market will likely reflect itself simultaneously in a $\beta_i \cdot x\%$ higher rate of return in your stock.
  - Together with alpha, beta can be interpreted as giving you the best conditional forecast of your project's rate of return, given a market outcome scenario's rate of return.
  - Practical estimation of future market-beta from historical stock return data is discussed in the book.
  - Yahoo!Finance lists estimates of betas for many stocks, too.

What is the market-beta of the market (the S&P500)?

Ceteris paribus, should/do investors prefer securities with a higher beta or a lower beta with respect to their (market) portfolio?

Should/do high beta or low beta projects have to offer higher average rates of return?

Should/do high variance or low variance projects have to offer higher average rates of return?

If you own a firm consisting of \$4 million invested in Division C, and \$6 million in Division D, what are this firm's returns?

	M	$M-\overline{M}$	C	D	Firm	$Fm-\overline{Fm}$
Yellow	−4	−5	−1.25	+3		
Red	−4	−5	+1.25	+13		
Green	+6	+5	+3.75	+3		
Blue	+6	+5	+1.25	−7		
Mean (E)	1	0	1.25	3		
Var	25	25	3.125	50		
$\beta_{i,M}$	1		0.25	−1		

Using the market's and your firm's rates of return, what is the market beta of your firm?

Is there a quicker way to compute the overall market-beta of your firm, based on its projects?

What statistics can you “value-average”? What statistics can you not “value-average.”

Hugely Important But Omitted

The mean-variance efficient frontier (almost equivalently, the mean-standard deviation efficient frontier).

- Covering it would require 1-2 full lectures. Though omitted, the mean-variance efficient frontier is extremely important. It is the basis for modern finance (and even for the CAPM, which goes too far to work well).
- In an Investments course (rather than a Corporate Finance course), you should be spending the time to work this out.
- The frontier gives you the optimal set of assets that you should hold if you want to tolerate a risk of x%. It also tells you what expected rate of return this portfolio (for your specific risk-tolerance) should give you.

The Sum of Variances

- There is a medium painful (but common and important) formula for computing the overall variance of a portfolio, based on the variance-covariance between all assets, and your investment in each asset. This formula makes it easy to recompute the portfolio risk when you change portfolio holdings. Its simplest form is for a portfolio P with two assets, A and B:

$$\mathrm{Var}(r_P) = \mathrm{Var}(w_A \cdot r_A + w_B \cdot r_B) =$$

$$w_A^2 \cdot \mathrm{Var}(R_A) + w_B^2 \cdot \mathrm{Var}(R_B) + 2 \cdot w_A \cdot w_B \cdot \mathrm{Cov}(R_A, R_B)$$

  This formula also shows that value weighting variances does not work, because this expression is *not* $w_A \cdot \mathrm{Var}(R_A) + w_B \cdot \mathrm{Var}(R_B)$.

What is the correlation of stocks' rates of returns from one day to the next day?

Time-Adjusting Risk:

If the risk of investing in the stock market for 1 year is $\sigma = 20\%$, what is the risk of investing for 10 years?

- There is an extremely important application.
- Let's assume that the per-unit-of-time standard deviation remains constant. Let's just call this number σ.
- Rates of return over time are usually uncorrelated (or you could use past stock returns to outpredict future stock returns). Algebraically,

$$\mathrm{Cov}(\mathrm{R_t}, \mathrm{R_{t+i}}) \approx 0$$

  where the subscripts t and $\mathrm{t+i}$ refer to time periods, not to stocks.

Continued

- In this case, the following approximation is not bad:

$$\boxed{\mathrm{Sdv}(\mathrm{R}_{0,\mathrm{T}}) \approx \sqrt{\mathrm{T}} \cdot \sigma}$$

  For example, if your portfolio risk is 10% per month, then your annual risk is about $\sqrt{12} \cdot 10\% \approx 35\%$ per year.
- The reason is that $\mathrm{Var}(\mathrm{R}_{0,\mathrm{T}}) \approx \mathrm{Var}(\mathrm{R}_{0,1} + \mathrm{R}_{1,2} + ... + \mathrm{R}_{\mathrm{T}-1,\mathrm{T}}) = \mathrm{Var}(\mathrm{R}_{0,1}) + \mathrm{Var}(\mathrm{R}_{1,2}) + ... + \mathrm{Var}(\mathrm{R}_{\mathrm{T}-1,\mathrm{T}}) + \text{many zero covariance terms} = \mathrm{T} \cdot \sigma$.
- This annualized sd is also used in the Sharpe-ratio, a (badly flawed but common) measure of investment performance that divides the historical average rate of return (net of the risk-free rate) by its standard deviation. (The SR of a portfolio grows with the square-root of time.)