3

Stock and Bond Valuation: Annuities and Perpetuities

Important Shortcut Formulas

The present value formula is the main workhorse for valuing investments of all types, including stocks and bonds. But these rarely have just two or three future payments. Stocks may pay dividends forever. The most common mortgage bond has 360 monthly payments. It would be possible but tedious to work with NPV formulas containing 360 terms.

Fortunately, there are some shortcut formulas that can speed up your PV computations if your projects have a particular set of cash flow patterns and the opportunity cost of capital is constant. The two most prominent are for projects called *perpetuities* (which have payments lasting forever) and *annuities* (which have payments lasting for a limited number of years). Of course, no firm lasts forever, but the perpetuity formula is often a useful "quick-and-dirty" tool for a good approximation. In any case, the formulas in this chapter are widely used and can help you understand the economics of corporate growth.

3.1 Perpetuities

A simple **perpetuity** is a project with a stream of constant cash flows that repeats forever. If the cost of capital (i.e., the appropriate discount rate) is constant and the amount of money remains the same or grows at a constant rate, perpetuities lend themselves to fast present value solutions—very useful when you need to come up with quick rule-of-thumb estimates. Though the formulas may seem intimidating at first, using them will quickly become second nature to you.

"Perpetuities" are projects with special kinds of cash flows, which permit the use of shortcut formulas.

The Simple Perpetuity Formula

At a constant interest rate of 10%, how much money do you need to invest today to receive the same dollar amount of interest of $2 each year, starting next year, forever? Exhibit 3.1 shows the present values of all future payments for a perpetuity paying $2 forever, if the interest rate is 10% per annum. Note how there is no payment at time 0, and that the individual payment terms become smaller and smaller the further out we go.

Here is an example of a perpetuity that pays $2 forever.

To confirm the table's last row, which gives the perpetuity's net present value as $20, you can spend from here to eternity to add up the infinite number of terms. But if you use a spreadsheet to compute and add up the first 50 terms, you will get a PV of $19.83. If you add up the first 100 terms, you will get a PV of $19.9986. Mathematically, the sum eventually converges to $20 sharp. This is because there is a nice shortcut to computing the net present value of the perpetuity if the cost of capital is constant:

The shortcut perpetuity formula.

Time	Cash Flow	Discount Factor	Present Value	Cumul PV
0	Nothing! You have no cash flow here!			
1	\$2	$1/(1+10\%)^1 \approx 0.909$	\$1.82	\$1.82
2	\$2	$1/(1+10\%)^2 \approx 0.826$	\$1.65	\$3.47
3	\$2	$1/(1+10\%)^3 \approx 0.751$	\$1.50	\$4.97
⋮	⋮	⋮	⋮	⋮
50	\$2	$1/(1+10\%)^{50} \approx 0.0085$	\$0.02	\$19.83
⋮	⋮	⋮	⋮	⋮
			Net Present Value (Sum):	\$20.00

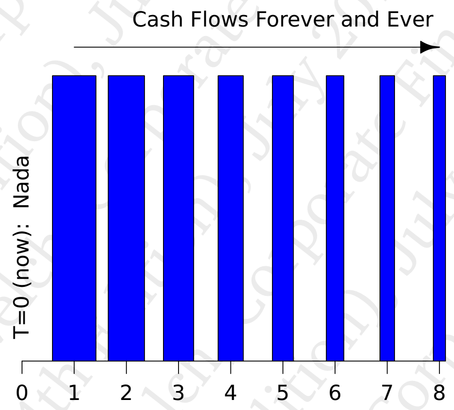

Exhibit 3.1: *Perpetuity Stream of \$2 with Interest Rate* $r = 10\%$. This exhibit shows cash flows, discount factors, and cumulative value. The height of the bars in the graph shows that the nominal cash flows are the same in every future period. Their widths (and thus their areas) indicate the present value of these cash flows. Each bar has less area than the preceding one. Otherwise, the cumulative sum could never be a finite number.

$$\text{Perpetuity PV} = \frac{\$2}{10\%} = \frac{\$2}{0.1} = \$20$$

$$PV_0 = \frac{C_1}{r}$$

The "1" time subscript in the formula is to remind you that the first cash flow occurs not now, but next year—the cash flows themselves will remain the same amount next year, the year after, and so on.

IMPORTANT

A stream of constant cash flows (C dollars each period and forever) beginning *next* period (i.e., time 1), which is discounted at the same per-period cost of capital r forever, is a special perpetuity worth

$$PV_0 = \frac{C_1}{r}$$

which is a shortcut for

$$PV_0 = \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + \frac{C_3}{(1+r)^3} + \cdots + \frac{C_t}{(1+r)^T} + \cdots$$

C_2 and all other C_t are the same as C_1.

The Oldest Institutions and Perpetuities

Perpetuities assume that projects last forever. But nothing does. The oldest Western institution today may well be the Roman Catholic Church. Wikipedia lists the oldest existing company as the Keiunkan hotel in Japan, founded in 705. (A number of existing restaurants, hotels, and breweries in the West are also fairly old, dating from the late ninth century.) The oldest existing corporation in the United States is the Collegiate Reformed Protestant Dutch Church of the City of New York, formed in 1628 and granted a corporate charter by King William in 1696. The Canadian Hudson's Bay Company was founded in 1670 and claims to be the oldest continuously *incorporated* company in the world. The oldest U.S. companies are the Stroh's brewery and the Bowne printing firm, both of which were founded in 1885.

Guantanamo Naval Base was leased from Cuba in 1903 as a perpetuity by the United States in exchange for 2,000 pesos per annum in U.S. gold, equivalent to $4,085. In a speech, Fidel Castro redefined time as "whatever is indefinite lasts 100 years." In any case, the Cuban government no longer recognizes the agreement and does not accept the annual payments—but it has also wisely not yet tried to expel the Americans. Let's see what diplomacy will do. *Wikipedia*

The easiest way for you to get comfortable with perpetuities is to solve some problems.

Easier done than said.

Q 3.1. From memory, write down the perpetuity formula. Be explicit on when the first cash flow occurs.

Q 3.2. What is the PV of a perpetuity paying $5 each month, beginning *next* month, if the monthly interest rate is a constant 0.5%/month?

Q 3.3. What is the PV of a perpetuity paying $15 each month, beginning *next* month, if the *effective* annual interest rate is a constant 12.68% per year?

Q 3.4. Under what interest rates would you prefer a perpetuity that pays $2 million per year beginning next year to a one-time payment of $40 million?

Q 3.5. In Britain, there are **Consol** bonds that are perpetuity bonds. (In the United States, the IRS does not allow companies to deduct the interest payments on perpetual bonds, so U.S. corporations do not issue Consol bonds.) What is the value of a Consol bond that promises to pay $2,000 per year if the prevailing interest rate is 4%?

The Growing Perpetuity Formula

A growing perpetuity assumes that cash flows grow by a constant rate g forever.

➤ Growing Perpetuities, Exhibit 3.2, Pg.40.

What if, instead of the same amount of cash every period, the cash flows increase over time? The **growing perpetuity** formula allows for a constant rate g per period, provided it is less than the interest rate. Exhibit 3.2 shows a growing perpetuity that pays \$2 next year, grows at a rate of 5%, and faces a cost of capital of 10%. The present value of the first 30 terms adds up to \$30.09. The first 100 terms add up to \$39.64. The first 200 terms add up to \$39.98. Eventually, the sum approaches the formula

$$\text{PV of Growing Perpetuity} = \frac{\$2}{10\% - 5\%} = \$40$$

$$PV_0 = \frac{C_1}{r - g} \tag{3.1}$$

Time	Cash Flow	Discount Factor	Present Value	Cumul PV
0	Nothing! You have no cash flow here!			
1	$(1+5\%)^0 \cdot \$2 = \$2.000$	$(1+10\%)^{-1} \approx 0.909$	\$1.818	\$1.82
2	$(1+5\%)^1 \cdot \$2 = \$2.100$	$(1+10\%)^{-2} \approx 0.826$	\$1.736	\$3.56
3	$(1+5\%)^2 \cdot \$2 = \$2.205$	$(1+10\%)^{-3} \approx 0.751$	\$1.657	\$5.22
⋮	⋮	⋮	⋮	⋮
30	$(1+5\%)^{29} \cdot \$2 \approx \$8.232$	$(1+10\%)^{-30} \approx 0.057$	\$0.472	\$30.09
⋮	⋮	⋮	⋮	⋮
		Net Present Value (Sum):		\$40.00

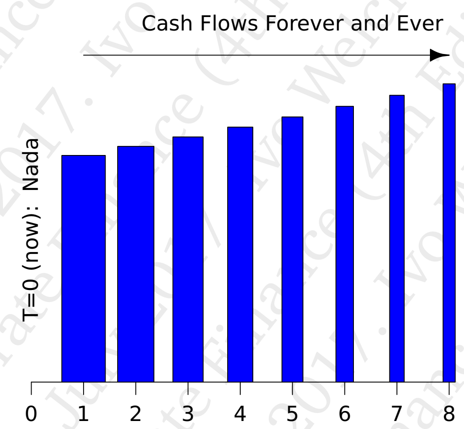

Exhibit 3.2: *Perpetuity Stream with* $C_1 = \$2$*, Growth Rate* $g = 5\%$*, and Interest Rate* $r = 10\%$. This exhibit shows cash flows, discount factors, and cumulative value. The height of the bars in the graph shows that the nominal cash flows are growing over time. However, their widths (and thus their areas) indicate the present value of these cash flows. Each bar has less area than the preceding one, which explains why the cumulative sum can be a finite number.

No cash flow at time 0. First growth is from time 1 to time 2.

As before, the "1" subscript indicates that cash flows begin next period, not this period, but here it is necessary because future cash flows will be different. The interest rate is r and it is reduced by g, the growth rate of your cash flows. Note how the table shows that the first application of the growth factor g occurs 1 period after the first application of the discount factor. For example, the cash flow at time 30 is discounted by $(1+r)^{30}$, but its cash flow is C multiplied by a growth factor of $(1+g)^{29}$. You will later encounter many applications of the growing perpetuity formula. For example, it is common to assume that cash flows grow by the rate of inflation. You will also later use this formula to obtain so-called terminal values in a chapter of this book, in which you design so-called pro formas.

➤ Terminal value, Sect. 21.2, Pg.585.

IMPORTANT

A stream of cash flows growing at a rate of g each period and discounted at a constant interest rate r is worth

$$PV_0 = \frac{C_1}{r - g}$$

The first cash flow, C_1, occurs next period (time 1), the second cash flow of $C_2 = C_1 \cdot (1 + g)$ occurs in two periods, and so forth, *forever*. For the formula to work, g can be negative, but r must be greater than g.

You need to memorize the growing perpetuity formula!

Be careful to use the cash flow *next* year in the numerator. The subscript "1" is there to remind you. For example, if you want to use this formula on your firm, and it earned $100 million this year, and you expect it to grow at a 5% rate forever, then the correct cash flow in the numerator is $C_1 = \$105$ million, not $100 million!

Although a subscript on C makes this seem more painful, it is a good reminder here.

What would happen if the cash flows grew faster than the interest rate ($g > r$)? Wouldn't the formula indicate a negative PV? Yes, but this is because the entire scenario would be nonsense. The present value in the perpetuities formulas is only less than infinity, because *in today's dollars*, each term in the sum is a little less than the term in the previous period. If g were greater than r, however, the cash flow 1 period later would be worth more even in today's dollars. For example, take our earlier example with a discount rate of 10%, but make the growth rate of cash flows $g = 15\%$. The first cash flow would still be $2, which still discounts to $1.818 today. But the second cash flow would be $\$2 \cdot 1.15 = \$2.30$, which discounts to $1.901 today. The third cash flow would be $\$2 \cdot 1.15^2 = \$2.645$, which discounts to $1.987 today. The present value of each cash flow is higher than that preceding it. Taking a sum over an infinite number of such increasing terms would yield infinity as the value. A value of infinity is clearly not sensible, as nothing in this world is worth an infinite amount of money. Therefore, the growing perpetuity formula yields nonsensical values if $g \geq r$—as it should!

The formula is nonsensical when $r < g$.

Q 3.6. From memory, write down the growing perpetuity formula.

Q 3.7. What is the PV of a perpetuity paying $5 each month, beginning *this* month (in 1 second), if the monthly interest rate is a constant 0.5%/ month (6.2%/year) and the cash flows will grow at a rate of 0.1%/month (1.2%/year)?

Q 3.8. What is the PV of a perpetuity paying $8 each month, beginning *this* month (in 1 second), if the monthly interest rate is a constant 0.5%/ month (6.2%/year) and the cash flows will grow at a rate of 0.8%/month (10%/year)?

Q 3.9. Here is an example of the most common use of the growing perpetuity model (called a pro forma). Your firm just finished the year, in which it had cash earnings of $100 million. Excluding this amount, you want to determine the value of the firm. You forecast your firm to have a quick growth phase for 3 years, in which it grows at a rate of 20% per annum (ending year 1 with $120 up to ending year 3 with $172.8). Your firm's growth then slows down to 10% per annum for the next 3 years (ending year 4 with $190.1, etc.). Finally, beginning in year 7, you expect it to settle into its long-term growth rate of 5% per annum. You also expect your cost of capital to be 10% in your 20% growth phase, 9% in your 10% growth phase, and 8% in your 5% growth phase. Excluding the $100 million, what do you think your firm is worth today?

Q 3.10. An eternal patent contract states that the patentee will pay the patentor a fee of $1.5 million next year. The contract terms state a fee growth with the inflation rate, which runs at 2% per annum. The appropriate cost of capital is 14%. What is the value of this patenting contract?

Q 3.11. How would the patent contract value change if the first payment did not occur next year, but tonight?

Application: Stock Valuation with A Gordon Growth Model

Perpetuities are imperfect approximations, but often give a useful upper bound.

With their fixed interest and growth rates and eternal payment requirements, perpetuities are rarely exactly correct. But they can be very helpful for quick back-of-the-envelope estimates. For example, consider a mature and stable business with profits of $1 million next year. Because it is stable, its profits are likely to grow at the inflation rate of, say, 2% per annum. This means that it will earn $1,020,000 in 2 years, $1,040,400 in 3 years, and so on. The firm faces a cost of capital of 8%. The growing perpetuity formula indicates that this firm should probably be worth no more than

$$\text{Business Value} = \frac{\$1{,}000{,}000}{8\% - 2\%} \approx \$16{,}666{,}667$$

$$\text{Business Value} = \frac{C_1}{r - g}$$

because in reality, the firm will almost surely not exist forever. Of course, in real life, there are often even more significant uncertainties: Next year's profit may be different, the firm may grow at a different rate (or may grow at a different rate for a while) or face a different cost of capital for 1-year loans than it does for 30-year loans. Thus, $16.7 million should be considered a quick-and-dirty useful approximation, perhaps for an upper limit, and not an exact number.

The Gordon growth model: constant eternal dividend growth.

The growing perpetuity model is sometimes directly applied to the stock market. For example, if you believe that a stock's dividends will grow by $g = 5\%$ forever, that the appropriate rate of return is $r = 10\%$, and that the stock market will earn and/or pay dividends of $D = \$10$ *next year*, then you would feel that a stock price today of

$$\text{Stock Price P Today} = \frac{\$10}{10\% - 5\%} = \$200$$

$$\text{Stock Price P Today} = \frac{\text{Dividends D Next Year}}{r - g} \tag{3.2}$$

would be appropriate. In this context, the growing perpetuity model is often called the **Gordon growth model**, after its inventor, Myron Gordon.

You could estimate the cost of capital for GE, based on its dividend yield and its expected dividend growth rate.

Let us explore the Gordon growth model a bit. In June 2016, YAHOO! FINANCE stated that General Electric (GE) had a dividend yield of 3.0%. This is the analysts' consensus forecast of next year's dividends divided by the stock price, D/P. This is called the **dividend yield**. Rearrange Formula 3.2:

$$\frac{\text{Dividends D Next Year}}{\text{Stock Price P Today}} = r - g = 3.0\%$$

Therefore, you can infer that the market believes that the appropriate cost of capital (r) for General Electric exceeds its growth rate of dividends (g) by about 3.0%. YAHOO! FINANCE further had a summary of GE's cash flow statement, which indicated that GE paid $9.3 billion in dividends in 2015, up 5% from 2014's $8.85 billion. Therefore, if you believe 5%/year is also a fair estimate of the *eternal* future growth rate of GE's dividends, then the financial markets valued GE as if it had a per-annum cost of capital of about

$$r = \frac{\text{Dividends D Next Year}}{\text{Stock Price P Today}} + g \approx 3\% + 5\% = 8\%$$

Don't take this estimate too seriously. It is an approximation that should be viewed just as a conversation starter.

Let's play another game that is prominent in the financial world. Earnings are, loosely speaking, cousins of the cash flows that corporate stockholders receive. You can then think of the value of the stock today as the value of the earnings stream the stock will produce. After all, recall from Chapter 1 that owners receive all dividends and all cash flows (earnings), presumably the former being paid out from the latter. (In Chapter 14, I will explain a lot of this in more detail as well as why earnings are only approximately but not exactly cash flows.)

Let's presume that the formula also applies to earnings.

Furthermore, it is common to assume that stock market values are capitalized as if corporate earnings were eternal cash flows that are growing at a constant rate g applicable to earnings (which is not necessarily the same as the growth rate applicable to dividends). This means that you would assume that the value of the firm is

You could also estimate the cost of capital for GE based on its price/earnings ratio and its earnings growth rate.

$$\text{Stock Price P Today} = \frac{\text{Earnings E Next Year}}{r - g}$$

Thus, to determine the rate of return that investors require (the cost of capital), all you need is a forecast of earnings, the current stock price, and the eternal growth rate of earnings. Again, YAHOO! FINANCE (Key Statistics and Analyst Estimates) gives you all the information you need. In June 2016, GE's "trailing P/E" ratio—calculated as the current stock price divided by historical earnings—was 41. More interestingly, the analysts predicted "forward P/E" ratios—calculated as the price divided by their expectations of *next* year's earnings—as 17. The growing perpetuity formula wants the earnings in *future* years, so the latter is closer to what you need. The analysts also expected GE's earnings to grow over the next 5 years at an average rate of 12%—the g in the formula if you are willing to assume that this is a long-term quasi-eternal growth rate. Therefore, all you have to do is rearrange the growing perpetuity formula, and out pops an appropriate rate of return:

$$r = \frac{\text{Earnings Next Year}}{\text{Stock Price Today}} + g = \frac{1}{\text{P/E}} + g \approx \frac{1}{17} + 12\% \approx 18\%$$

As a herd, analysts were quite optimistic on GE's earnings relative to its price and more so than they were with respect to how much it would pay out in dividends.

This formula is intuitive, but there are more complex versions. For example, analysts sometimes use one that contemplates that firms with higher earnings reinvestment rates (aka **plowback** ratios) should have higher earnings growth rates g.

Mention plowback extensions

It is important that you recognize these are just approximations that you should not take too seriously in terms of accuracy. GE will not last forever, earnings are not the cash flows you need, the discount rate is not eternally constant, earnings will not grow forever at 6.3%, and so on. However, the numbers are not uninteresting and may not even be too far off, either. GE is a very stable company that is likely to be around for a long time, and you could do a lot worse than assuming that the cost of capital (for investing in projects that are similar to GE stock ownership) is somewhere around 12% per annum—say, somewhere between 10% to 14% per annum.

Keep perspective! The model provides only a quick approximation.

➤ Price-earnings ratio, Sect. 15.3, Pg.392.

Q 3.12. A stock is paying a quarterly dividend of $5 in 1 month. The dividend is expected to increase every quarter by the inflation rate of 0.5% per quarter—so it will be $5.025 in the next quarter (i.e., paid out in 4 months). The prevailing cost of capital for this kind of stock is 9% per annum. What should this stock be worth?

Q 3.13. If a $100 stock has earnings of $5 per year, and the appropriate cost of capital for this stock is 12% per year, what does the market expect the firm's "as-if-eternal dividends" to grow at?

3.2 Annuities

An annuity pays the same amount for T years.

The second type of cash flow stream that lends itself to a quick formula is an **annuity**, which is a stream of equal cash flows for a given number of periods. Unlike a perpetuity, payments stop after T periods. For example, if the interest rate is 10% per period, what is the value of an annuity that pays $5 per period for 3 periods?

By hand.

Let's first do this the slow way. You can hand-compute the net present value as

$$PV = \frac{\$5}{1.10} + \frac{\$5}{1.10^2} + \frac{\$5}{1.10^3} \approx \$12.4343$$

$$PV = \frac{C_1}{(1+r_1)} + \frac{C_2}{(1+r_2)} + \frac{C_3}{(1+r_3)}$$

$$= \frac{C_1}{(1+r)} + \frac{C_2}{(1+r)^2} + \frac{C_3}{(1+r)^3}$$

The annuity formula makes short work of this NPV calculation,

$$PV = \$5 \cdot \left\{\frac{1-[1/(1+10\%)]^3}{10\%}\right\} \approx \$12.4343$$

$$PV = C_1 \cdot \left\{\frac{1-[1/(1+r)]^T}{r}\right\} = PV$$

Is this really a shortcut? Maybe not for 3 periods, but try a 360-period annuity—which method do you prefer? Either works.

IMPORTANT

A stream of constant equal cash flows, beginning next period (time 1) and lasting for T periods, and discounted at a constant interest rate r, is worth

$$PV_0 = \frac{C_1}{r} \cdot \left[1 - \frac{1}{(1+r)^T}\right]$$

Q 3.14. How many years does it take for an annuity to reach three-quarters the value of a perpetuity if the interest rate is 5%? If the interest rate is r? To reach fraction f of the value?

Q 3.15. Recall from memory the annuity formula.

Q 3.16. What is the PV of a 360-month annuity paying $5 per month, beginning at $5 next month (time 1), if the monthly interest rate is a constant 0.5%/month (6.2%/year)?

Q 3.17. In *L'Arithmetique*, written in 1558, Jean Trenchant posed the following question: "In the year 1555, King Henry, to conduct the war, took money from bankers at the rate of 4% per fair [quarter]. That is better terms for them than 16% per year. In this same year before the fair of Toussaints, he received by the hands of certain bankers the sum of 3,945,941 ecus and more, which they called 'Le Grand Party' on the condition that he will pay interest at 5% per fair for 41 fairs after which he will be finished. Which of these conditions is better for the bankers?" Translated, the question is whether a perpetuity at 4% interest payout per quarter is better or worse than a 41-quarter annuity at 5% interest payout per quarter. (The answer will depend on the prevailing true interest rate, which you can assume to be constant.)

Q 3.18. Solve Fibonacci's annuity problem given in the anecdote on the next page: Compare the PV of a stream of quarterly cash flows of 75 bezants versus the PV of a stream of annual cash flows of 300 bezants. Payments are always at period-end. The interest rate is 2% per month. What is the relative value of the two streams? Compute the difference for a 1-year investment first.

Fibonacci and the Invention of Net Present Value

William Goetzmann argues that Leonardo of Pisa, commonly called Fibonacci, may have invented not only the famous "Fibonacci series" but also the concept of net present value.

Fibonacci's family were merchants in the Mediterranean in the thirteenth century, with trade relations to Arab merchants in Northern Africa. Fibonacci wrote about mathematics primarily as a tool to solve merchants' problems—in effect, to understand the pricing of goods and currencies relative to one another. Imagine how rich you could get if you were the only one who could quickly determine which goods were worth more than others! In fact, you should think of Fibonacci and other Pisan merchants as the "financial engineers" of the thirteenth century.

In 1202, the 30-year-old Fibonacci published his most famous treatise, *Liber Abaci*. We still are using its problems and answers today. One of his puzzles—which you solve in Q3.17—is called "On a Soldier Receiving 300 Bezants for His Fief":

> A soldier is granted an annuity by the king of 300 bezants per year, paid in quarterly installments of 75 bezants. The king alters the payment schedule to an annual year-end payment of 300. The soldier is able to earn 2 bezants on 100 per month (over each quarter) on his investment. How much is his effective compensation after the terms of the annuity changed?

To answer this problem, you must know how to value payments at different points in the future—you must understand the time value of money. What is the value of 75 bezants in one quarter, two quarters, and so forth? What is the value of 300 bezants in one year, two years, and so on? Yes, money sooner is usually worth more than money later—but you need to determine by exactly how much in order to determine how good or bad the change is for the king and the soldier. You must use the interest rate Fibonacci gives and then compare the two different cash flow streams—the original payment schedule and the revised payment schedule—in terms of a common denominator. This common denominator will be the two streams' present values.

William Goetzmann, Yale University

Annuity Application: Fixed-Rate Mortgage Payments

Most mortgages are **fixed-rate mortgage loans**, and they are basically annuities. They promise a specified stream of equal cash payments each month to a lender. A 30-year mortgage with monthly payments is really a 360-payment annuity. (The "annu-ity" formula should really be called a "month-ity" formula in this case.) What would be your monthly payment if you took out a 30-year mortgage loan for \$500,000 at a quoted interest rate of 7.5% per annum?

Mortgages and other loans are annuities, so the annuity formula is in common use.

Before you can proceed further, you need to know one more bit of institutional knowledge here: Mortgage providers—like banks—quote interest by just dividing the mortgage quote by 12, so the true monthly interest rate is $7.5\%/12 = 0.625\%$. (They do not compound; if they did, the monthly interest rate would be $(1+7.5\%)^{1/12} - 1 \approx 0.605\%$.)

Lenders quote interest rates using the same convention as banks.

A 30-year mortgage is an annuity with 360 equal payments with a discount rate of 0.625% per month. Its PV of \$500,000 is the amount that you are borrowing. You want to determine the fixed monthly cash flow that gives the annuity this value:

The mortgage payment can be determined by solving the annuity formula.

$$
\begin{aligned}
\$500{,}000 &= \frac{C_1}{0.625\%} \cdot \left[1 - \frac{1}{(1+0.625\%)^{360}}\right] \approx C_1 \cdot 143.018 \\
PV &= \frac{C_1}{r} \cdot \left[1 - \frac{1}{(1+r)^T}\right]
\end{aligned}
$$

Solving for the cash flow tells you that the monthly payment on your $500,000 mortgage will be $500,000/143.018 ≈ $3,496.07 for 360 months, beginning next month (time 1).

Principal and Interest Components

Repayment and taxes are reasons to determine principal and interest components.

There are two reasons why you may want to determine how much of your $3,496.07 payment should be called interest payment and how much should be called principal repayment. The first reason is that you need to know how much principal you owe if you want to repay your loan early. The second reason is that Uncle Sam allows mortgage borrowers to deduct the interest, but not the principal, from their tax bills.

Principal repayment is the sum left over after the interest payment from your monthly installment.

Here is how you can determine the split: In the first month, you pay $0.625\% \cdot \$500{,}000 = \$3{,}125$ in mortgage interest. Therefore, the principal repayment is $\$3{,}496.07 – \$3{,}125 = \$371.07$ and the remaining principal is $499,628.93. The following month, your interest payment is $0.625\% \cdot \$499{,}628.93 \approx \$3{,}122.68$ (note that your interest payment is now on the remaining principal), which leaves $\$3{,}496.07 - \$3{,}122.68 = \$373.39$ as your principal repayment, and $499,255.54 as the remaining principal. And so on.

Q 3.19. Rental agreements are not much different from mortgages. For example, what would your rate of return be if you rented your $500,000 warehouse for 10 years at a monthly lease payment of $5,000? If you can earn 5% per annum elsewhere, would you rent out your warehouse?

Q 3.20. What is the monthly payment on a 15-year mortgage for every $1,000 of mortgage at an effective interest rate of 6.168% per year (here, 0.5% per month)?

Application: A Level-Coupon Bond

Unlike zero-bonds, coupon bonds pay not only at the final time but also at interim points in time.

Let us exercise your newfound knowledge in a more elaborate example—this time with bonds. Recall that a bond is a financial claim sold by a firm or government. Bonds come in many varieties, but one useful classification is into coupon bonds and zero-bonds (short for **zero coupon bonds**). A **coupon bond** pays its holder cash at many different points in time, whereas a **zero-bond** pays only a single lump sum at the maturity of the bond with no interim coupon. Many coupon bonds promise to pay a regular coupon similar to the interest rate prevailing at the time of the bond's original sale, and then return a "principal amount" plus a final coupon at the end of the bond.

Bond naming conventions specify their promised payout patterns.

For example, think of a coupon bond that will pay $1,500 each half-year (semi-annual payment is very common) for 5 years, plus an additional $100,000 in 5 years. This payment pattern is so common that it has specially named features: A bond with coupon payments that remain the same for the life of the bond is called a **level-coupon bond**. These are the most common bonds today. The $100,000 here would be called the **principal**, in contrast to the $1,500 semiannual coupon. Level bonds are commonly named by just adding up all the coupon payments over 1 year (here, $3,000) and dividing this sum of annual coupon payments by the principal. Thus, this particular bond would be called a "3% semiannual coupon bond" ($3,000 coupon per year divided by the principal of $100,000). Now, the "3% coupon bond" is just a naming convention for the bond with these specific cash flow patterns—it is not the interest rate that you would expect if you bought this bond. In Section 2.4, we called such name designations interest *quotes*, as distinct from interest *rates*.

➤ Compounding, Sect. 2.4, Pg.16.

Step 1: Write down the bond's payment stream.

What should this $100,000, 3% semiannual level-coupon bond sell for today? First, you should write down the payment structure for a 3% semiannual coupon bond. This comes from its defined promised payout pattern:

Year	Due Date	Bond Payment	Year	Due Date	Bond Payment
0.5	Nov 2016	$1,500	3.0	May 2019	$1,500
1.0	May 2017	$1,500	3.5	Nov 2019	$1,500
1.5	Nov 2017	$1,500	4.0	May 2020	$1,500
2.0	May 2018	$1,500	4.5	Nov 2020	$1,500
2.5	Nov 2018	$1,500	5.0	May 2021	$101,500

Second, you need to determine the appropriate rates of return that apply to these cash flows. In this example, assume that the prevailing interest rate is 5% per annum. This translates into 2.47% for 6 months, 10.25% for 2 years, and so on.

Step 2: Find the appropriate cost of capital for each payment.

Year	Maturity	Discount Rate	Year	Maturity	Discount Rate
0.5	6 Months	2.47%	3.0	36 Months	15.76%
1.0	12 Months	5.00%	3.5	42 Months	18.62%
1.5	18 Months	7.59%	4.0	48 Months	21.55%
2.0	24 Months	10.25%	4.5	54 Months	24.55%
2.5	30 Months	12.97%	5.0	60 Months	27.63%

Third, compute the discount factors, which are just $1/(1+r_t) = 1/(1+r)^t$, and multiply each future payment by its discount factor. This will give you the present value (PV) of each bond payment. From there, you can compute the bond's overall value:

Step 3: Compute the discount factor $1/(1+r_t)$.

Year	Due Date	Bond Payment	Rate of Return	Discount Factor	Present Value
0.5	Nov 2016	$1,500	2.47%	0.9759	$1,463.85
1.0	May 2017	$1,500	5.00%	0.9524	$1,428.57
1.5	Nov 2017	$1,500	7.59%	0.9294	$1,394.14
2.0	May 2018	$1,500	10.25%	0.9070	$1,360.54
2.5	Nov 2018	$1,500	12.97%	0.8852	$1,327.76
3.0	May 2019	$1,500	15.76%	0.8638	$1,295.76
3.5	Nov 2019	$1,500	18.62%	0.8430	$1,264.53
4.0	May 2020	$1,500	21.55%	0.8277	$1,234.05
4.5	Nov 2020	$1,500	24.55%	0.8029	$1,204.31
5.0	May 2021	$101,500	27.63%	0.7835	$79,527.91
				Sum:	$91,501.42

You now know that you would expect this 3% semiannual level-coupon bond to be trading for $91,501.42 today in a perfect market. Because the current price of the bond is below its named final principal payment of $100,000, this bond would be said to trade at a **discount**. (The opposite would be a bond trading at a **premium**.)

Discount and premium bonds.

The bond's value can be calculated more quickly via the annuity formula. Let's work in half-year periods. You have 10 coupon cash flows, each $1,500, at a per-period interest rate of 2.47%. According to the formula, these 10 coupon payments are worth

Using the annuity formula to speed your calculations.

$$\text{PV} = C_1 \cdot \left\{ \frac{1-[1/(1+r)]^T}{r} \right\} = \$1{,}500 \cdot \left\{ \frac{1-[1/(1.0247)]^{10}}{2.47\%} \right\} \approx \$13{,}148.81$$

In addition, you have the $100,000 repayment of principal, which will occur in year 5 and is therefore worth

$$\text{PV} = \frac{\$100{,}000}{(1+5\%)^5} \approx \frac{\$100{,}000}{1+27.63\%} \approx \$78{,}352.62$$

$$\text{PV} = \frac{C_5}{(1+r)^5} = \frac{C_5}{(1+r_5)}$$

Together, the present values of the bond's cash flows again add up to $91,501.42.

The coupon rate is not the interest rate.

Important Reminder of Quotes versus Returns: Never confuse a bond designation with the interest it pays. The "3% semiannual coupon bond" is just a designation for the bond's payout pattern. The bond will not give you coupon payments equal to 1.5% of your $91,501.42 investment (which would be $1,372.52). The prevailing interest rate (cost of capital) has nothing to do with the quoted interest rate on the coupon bond. You could just as well determine the value of a 0% coupon bond, or a 10% coupon bond, given the prevailing 5% economy-wide interest rate. Having said all this, in the real world, many corporations choose coupon rates similar to the prevailing interest rate, so that at the moment of inception, the bond will be trading at neither a premium nor a discount. At least for this one brief at-issue instant, the coupon rate and the economy-wide interest rate may actually be fairly close. However, soon after issuance, market interest rates will move around, while the bond's payments will remain fixed, as designated by the bond's coupon name.

Q 3.21. You already learned that the value of one fixed future payment and the interest rate move in opposite directions (Page 25). What happens to the bond price of $91,501.42 in the level-coupon bond example if the economy-wide interest rates were to suddenly move from 5% per annum to 6% per annum?

Q 3.22. Assume that the 3% level-coupon bond discussed in this chapter has not just 5 years with 10 payments, but 20 years with 40 payments. Also, assume that the interest rate is not 5% per annum, but 10.25% per annum. What are the bond payment patterns and the bond's value?

Q 3.23. Check that the rates of return in the coupon bond valuation example on Page 47 are correct.

3.3 The Four Formulas Summarized

The growing annuity formula—it is used only rarely.

I am not a fan of memorization, but you must remember the growing perpetuity formula. You must also remember the annuity formula. They are used in many different contexts. There is also a **growing annuity** formula, which nobody remembers, but which you can look up if you need it:

$$\text{PV} = \frac{C_1}{r-g} \cdot \left[1 - \frac{(1+g)^T}{(1+r)^T}\right] \tag{3.3}$$

It is sometimes used in the context of pension cash flows, which tend to grow for a fixed number of time periods (T in the formula above) and then stop. However, even then it is not a necessary device. It is often more convenient and flexible to just work with the cash flows themselves within a spreadsheet.

A full summary.

Exhibit 3.3 summarizes the four special cash flows. The top graph shows the pattern of cash flows. For perpetuities, they go on forever. For annuities, they stop eventually. The bottom graph shows the present value of these cash flows. Naturally, these bars are shorter than those of their cash flows, which just means that there is a time value of money. The applicable formulas are below the graphs.

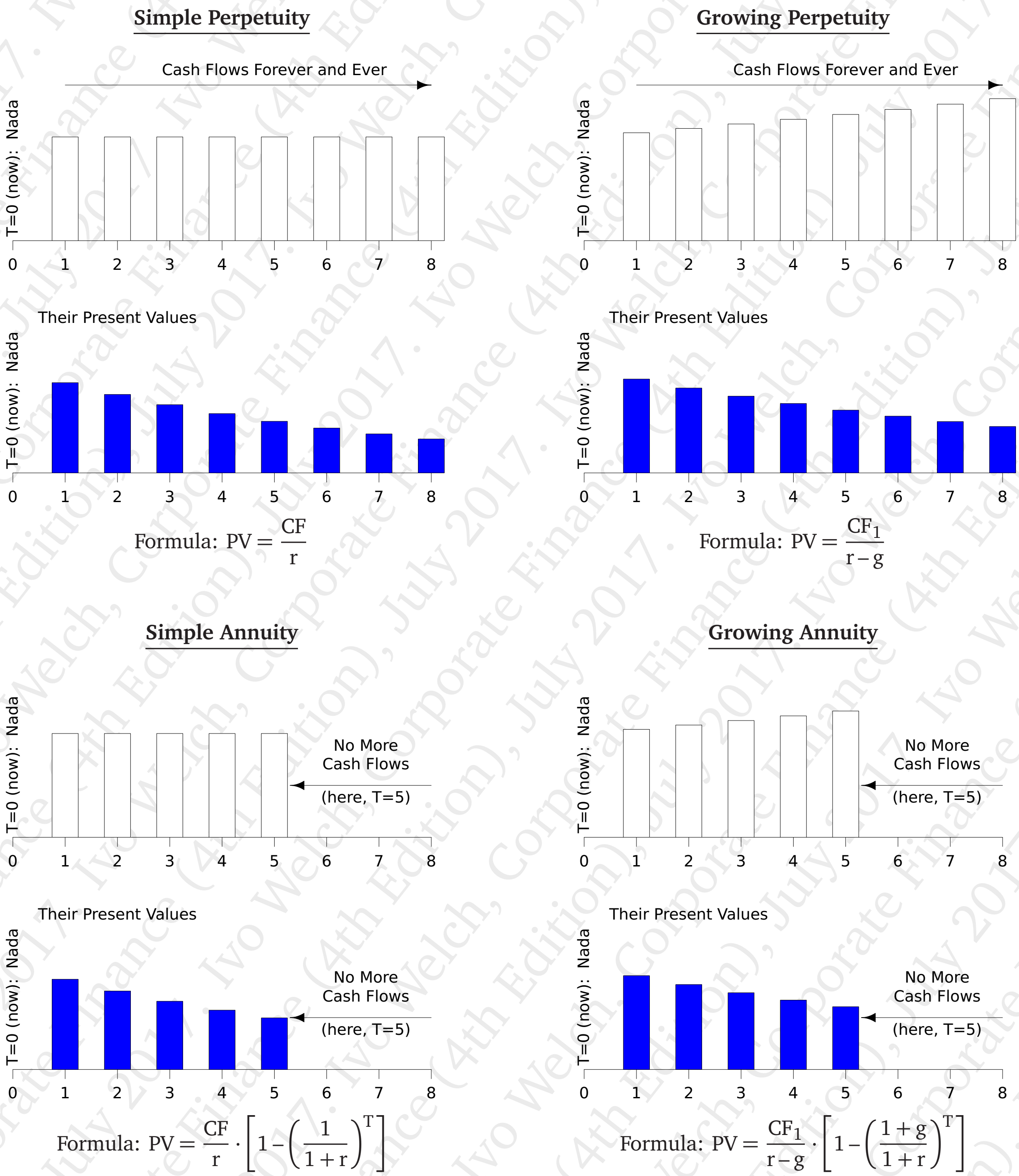

Exhibit 3.3: *The Four Payoff Streams and Their Present Values.*

Q 3.24. In many defined-contribution pension plans, the employer provides a fixed-percentage contribution to the employee's retirement. Assume that the employer must contribute $4,000 per annum beginning next year (time 1), growing annually with the inflation rate of 2% per year. What is the present value of the pension cost of hiring a 25-year-old who will stay with the company for 35 years? Assume a discount rate of 8% per year. Note: Please look up the growing annuity formula to solve this problem.

Summary

This chapter covered the following major points:

- Exhibit 3.3 summarizes the four special cash flows and their quick valuation formulas.
- The PV of a simple perpetuity, which is a stream of constant cash flows that begin next period and that are to be discounted at the same annual cost of capital forever, is

$$PV = \frac{C_1}{r}$$

- The PV of a growing perpetuity—with constant growth g, cash flows C beginning next year (time 1), and constant per-period interest rate r—is

$$PV = \frac{C_1}{r - g}$$

- Stocks are often valued through an application of the growing perpetuity formula, called the Gordon dividend growth model.
- The PV of an annuity—T periods of constant C cash flows (beginning next year) and constant per-period interest rate r—is

$$PV = C_1 \cdot \left\{ \frac{1 - [1/(1 + r)]^T}{r} \right\}$$

- Fixed-rate mortgages are annuities. Interest rate quoted on such bonds are computed with the annuity formula.

Preview of the Chapter Appendix in the Companion

The appendix to this chapter in the companion (not here)

- shows how the annuity and perpetuity formulas can be derived.
- explains "equivalent annual costs" (which you already briefly encountered in Question 2.39). These allow you to compare projects with different rental periods—such as an 8-year lease that charges $1,000 per year and a 10-year lease that charges $900 per year.

➤ Equivalent Annual Cost, Question 2.39, Pg.29.

Keywords

Answers

Q 3.1 C_1/r. The first cash flow occurs next period, not this period.

Q 3.2 $PV = C_1/r = \$5/0.005 = \$1{,}000$

Q 3.3 The interest rate is $1.1268^{(1/12)} - 1 \approx 1\%$ per month. Thus, $PV = C_1/r \approx \$15/0.01 \approx \$1{,}500$.

Q 3.4 Rearrange $P = C_1/r$ into $r = C_1/P = \$2/\$40 = 5\%$. At a 5% interest rate, you are indifferent. If the interest rate is above 5%, the immediate one-time payment is better, because future cash flows are less valuable. If the interest rate is below 5%, the perpetuity payment is better, because future cash flows are more valuable.

Q 3.5 $PV = \$2{,}000/4\% = \$50{,}000$

Q 3.6 $C_1/(r-g)$.

Q 3.7 You get $C_0 = \$5$ today, and next month you will receive a payment of $C_1 = (1+g)\cdot C_0 = 1.001\cdot\$5 = \$5.005$. The growing perpetuity is worth $PV = C_1/(r-g) = \$5.005/(0.5\% - 0.1\%) = \$1{,}251.25$. The total value is \$1,256.25.

Q 3.8 This is a nonsensical question, because the value would be infinite if $g \geq r$.

Q 3.9 Your earnings will be as follows:

Yr	Earn Was	Grwth Rate	Earn Is	Yearly Dscnt R	Comp. Dscnt R	Present Value
0-1	\$100	20%	\$120.0	10%	10%	\$109.09
1-2	\$120	20%	\$144.0	10%	21%	\$119.01
2-3	\$144	20%	\$172.8	10%	33.1%	\$129.83
3-4	\$172.8	10%	\$190.1	9%	45.1%	\$131.02
4-5	\$190.1	10%	\$209.1	9%	58.1%	\$132.22
5-6	\$209.1	10%	\$230.0	9%	72.4%	\$133.43
6-7	\$230.0	5%	\$241.5	8%	86.2%	\$129.73
7-8	\$241.5	5%	\$253.6	8%	...	...
⋮	⋮	⋮	⋮	⋮	⋮	⋮
				PV (Yr 0 through 7):		\$884.33
Subseq Perp in Yr 7:	\$8,452.38				PV Perp	\$4,540.44
				Total Project PV:		\$5,424.76

Standing in year 7, the growing perpetuity with cash flows of \$253.6 (projected for year 8) is worth $\$253.6/(8\% - 5\%) \approx \$8{,}452$. (If you want, you could round this number to \$8,500. If you are concerned about my rounding too aggressively, you have lost perspective—there is no firm in this world for which you can forecast the value in eight years with this much accuracy!) The \$8.453 billion is our assumption of what we will be able to sell the firm for at the end of year 7. It is our **terminal value**. All cash flows in year 7 (both the \$241.50 that we will still take home in year 7, plus the \$8,452) must then be discounted by 86.2%. Therefore, the PV is about \$884 million from cash flows that you computed explicitly (years 1 through 7), plus $\$8,452) \approx \$4.540$ billion from the cash flows that is the terminal value stand-in for all cash flows from year 8 to infinity. In sum, the estimate of this firm's present value is around \$5.4 billion. (Note: You could also calculate a terminal value in year 6 (for year 7 and beyond), and reach the same answer.)

Q 3.10 $\$1.5 \text{ million}/(14\% - 2\%) = \$12.5$ million.

Q 3.11 The immediate dividend would be worth \$1.5 million. In addition, you now have a growing perpetuity that starts with a payment of \$1.530 million. Therefore, the PV would be $\$1.500 + \$1.530/12\% = \$14.250$ million. Alternatively, you could multiply the \$12.5 million from your answer to Question 3.10 by $(1 + 14\%)$.

Q 3.12 First work out what the value would be if you stood at 1 month. The interest rate is $(1+9\%)^{1/12} - 1 \approx 0.7207\%$ per month, and $1.007207^3 - 1 \approx 2.1778\%$ per quarter. Thus, in 1 month, you will be entitled to a dividend stream of $\$5.025/(2.1778\% - 0.5\%) \approx \$299.50$. In addition, you get the \$5 for a total of \$304.50. Because this is your value in 1 month, discount \$304.50 at a 0.7207% interest rate to \$302.32 today.

Q 3.13 $g = r - E/P = 12\% - \$5/\$100 = 7\%$ per annum

Q 3.14 Compare the annuity and perpetuity formulas. The difference between them is the $1 - 1/(1+r)^t$ term. To be three-quarters of the value, this term has to be 3/4. So you must solve $1 - 1/(1+r)^t = 3/4$, or $1/(1+r)^t = 1 - 3/4 = 1/4$ or $(1+r)^t = 4$. Taking logs, $t = \log(4)/\log(1+r)$. In the main question, r was 5%, so $t = \log(4)/\log(1.05) \approx 28.41$ years. More generally, to reach a given fraction f of value, $t = \log[1/(1-f)]/\log(1+r)$. Think of this number of years as helping you judge the quality of the infinite-period approximation in the real world. If it is more realistic that you have fewer than 30 years of cash flows instead of an infinite stream, then the perpetuity formula may not be a great approximation of value when the interest rate is 5%.

Q 3.15 The annuity formula is $C_1 \cdot \left(\{1 - [1/(1+r)]^T\}/r\right)$.

Q 3.16 Your 360-month annuity is worth

$$C_1 \cdot \left\{\frac{1 - [1/(1+r)]^T}{r}\right\} = \$5 \cdot \left\{\frac{1 - [1/(1+0.005)]^{360}}{0.005}\right\}$$
$$\approx \$5 \cdot \left\{\frac{1 - 0.166}{0.005}\right\} \approx \$833.96$$

Q 3.17 The unknown interest rate is r. For each 1e that you have lent out, if you have a choice of 0.04e payment forever or a 0.05e payment for 41 months, you would compare 0.04e/r to $(0.05e/r)\cdot[1-1/(1+r)^{41}]$. If $r<0.0400352$, you would prefer the perpetuity. Otherwise, you prefer the annuity.

Q 3.18 For one year, the 300 bezants paid once at year-end are worth $300b/1.02^{12} \approx 236.55$ bezants today. Now for the quarterly payment schedule: The quarterly interest rate is $1.02^3-1\approx 6.12\%$. Therefore, the 4-"quartity" is worth $75b/0.0612\cdot[1-1/1.0612^4]\approx 75b/1.0612^1+75b/1.0612^2+75b/1.0612^3+75b/1.0612^4 \approx 259.17$ bezants. The soldier would have lost 22.62 bezants in present value, which is 8.73% of what he was promised. (The same loss of $236.55/259.17-1\approx 8.73\%$ would apply to longer periods.)

Q 3.19 To find the implicit cost of capital of the lease, you need to solve

$$\$500{,}000 = \frac{\$5{,}000}{r}\cdot\left[1-\frac{1}{(1+r)^{120}}\right]$$

The solution is $r\approx 0.31142\%$ per month, or 3.8% per annum. This is the implied rate of return if you buy the warehouse and then rent it out. You would be better off earning 5% elsewhere.

Q 3.20 For \$1,000 of mortgage, solve for C_1 in

$$\begin{aligned} \text{PV} &= C_1\cdot\left\{\frac{1-[1/(1+r)]^T}{r}\right\} \\ \$1{,}000 &= C_1\cdot\left\{\frac{1-[1/(1.005)]^{15\cdot 12=180}}{0.005}\right\} \approx C_1\cdot 118.504 \\ \iff\quad & C_1 \approx \$8.44 \end{aligned}$$

In other words, for every \$1,000 of loan, you have to pay \$8.44 per month. For other loan amounts, just rescale the amounts.

Q 3.21 The semiannual interest rate would now increase from 2.47% to

$$r = \sqrt[2]{1+6\%}-1 = \sqrt{1.06}-1 \approx 2.9563\%$$

To get the bond's new present value, reuse the annuity formula

$$\begin{aligned} \text{PV} &= C_1\cdot\left\{\frac{1-[1/(1+r)]^T}{r}\right\} + \frac{C_t}{1+r_t} \\ &\approx \$1{,}500\cdot\left\{\frac{1-[1/(1+2.9563\%)]^{10}}{2.9563\%}\right\} + \frac{\$100{,}000}{(1+2.9563\%)^{10}} \\ &\approx \$12{,}823.89 + \$74{,}725.82 \\ &\approx \$87{,}549.70 \end{aligned}$$

This bond would have lost \$3,951.72, or 4.3% of the original investment.

Q 3.22 The interest rate is 5% per half-year. Be my guest if you want to add 40 terms. I prefer the annuity method. The coupons are worth

$$\begin{aligned} \text{PV}(\text{Coupons}) &= C_1\cdot\left\{\frac{1-[1/(1+r)]^T}{r}\right\} \\ &= \$1{,}500\cdot\left\{\frac{1-[1/(1.05)]^{40}}{0.05}\right\} \approx \$25{,}738.63 \end{aligned}$$

The final payment is worth PV(Principal Repayment) $= \$100{,}000/(1.05)^{40} \approx \$14{,}204.57$. Therefore, the bond is worth about \$39,943.20 today.

Q 3.23 For 6 months, $(1+2.47\%)^2-1\approx 5\%$. Now, define 6 months to be 1 period. Then, for t 6-month periods, you can simply compute an interest rate of $(1+2.47\%)^t-1$. For example, the 30 months interest rate is $1.0247^5-1\approx 12.97\%$.

Q 3.24 The solution is $\$4{,}000/(0.08-0.02)\cdot\left[1-\frac{1.02^{35}}{1.08^{35}}\right]\approx$ \$57,649.23.

End of Chapter Problems

Q 3.25. A tall Starbucks coffee costs \$1.85 in 2017. If the bank's quoted interest rate is 6% per annum, compounded daily, and if the Starbucks price never changed, what would an endless, inheritable free subscription to one Starbucks coffee per day be worth today?

Q 3.26. If you could pay for your mortgage forever, how much would you have to pay per month for a \$1,000,000 mortgage, at a 6.5% annual interest rate? Work out the answer (a) if the 6.5% is a bank APR quote and (b) if the 6.5% is a true effective annual rate of return.

Q 3.27. What is the PV of a perpetuity paying \$30 each month, beginning *next* month, if the annual interest rate is a constant effective 12.68% per year?

Q 3.28. What is the prevailing interest rate if a perpetual bond were to pay \$100,000 per year *beginning next year* and costs \$1,000,000 today?

Q 3.29. What is the prevailing interest rate if a perpetual bond were to pay \$100,000 per year *beginning next year* (time 1) and payments grow with the inflation rate at about 2% per year, assuming the bond costs \$1,000,000 today?

Q 3.30. A tall Starbucks coffee costs \$1.85 a day. If the bank's quoted interest rate is 6% per annum and coffee prices increased at a 3% annual rate of inflation, what would an endless, inheritable free subscription to one Starbucks coffee per day be worth today?

Q 3.31. Economically, why does the growth rate of cash flows have to be less than the discount rate?

Q 3.32. Your firm just finished the year, in which it had cash earnings of \$400. You forecast your firm to have a quick growth phase from year 0 to year 5, in which it grows at a rate of 40% per annum. Your firm's growth then slows down to 20% per annum between year 5 to year 10. Finally, beginning in year 11, you expect the firm to settle into its long-term annual growth rate of 2% . You also expect your cost of capital to be 15% over the first 5 years, then 10% over the next 5 years, and 8% thereafter. What do you think your firm is worth today? (Advice: Use a computer spreadsheet program.)

Q 3.33. A stock pays an annual dividend of \$2. The dividend is expected to increase by 2% per year (roughly the inflation rate) forever. The price of the stock is \$40 per share. At what cost of capital is this stock priced?

Q 3.34. A tall Starbucks coffee costs \$1.85 a day. If the bank's quoted interest rate is 6% per annum, compounded daily, and if the Starbucks price never changed, what would a lifetime free subscription to one Starbucks coffee per day be worth today, assuming you will live for 50 more years? What should it be worth to you to be able to bequeath or sell it upon your departure?

Q 3.35. What maximum price would you pay for a standard 8% level-coupon bond (with semiannual payments and a face value of \$1,000) that has 10 years to maturity if the prevailing discount rate (your cost of capital) is an effective 10% per annum?

Q 3.36. If you have to pay off an effective 6.5% loan within the standard 30 years, then what are the per-month payments for the \$1,000,000 mortgage? As in Question 3.26, consider both an effective 6.5% interest rate per year, and a bank quote of 6.5% (APR) per year.

Q 3.37. Structure a mortgage bond for \$150,000 so that its monthly payments are \$1,000. The prevailing interest rate is quoted at 6% (APR) per year.

Q 3.38. (Advanced) You are valuing a firm with a "pro forma" (i.e., with your forward projection of what the cash flows will be). The firm just had cash flows of \$1,000,000 today. This year, it will be growing by a rate of 20% per annum. That is, at the end of year 1, the firm will have a cash flow of \$1.2 million. In each of the following years, the difference between the growth rate and the inflation rate of 2% will (forever) halve. Thus, from year 1 to year 2, the growth rate will be $2\% + (20\% - 2\%)/2 = 11\%$, so the next cash flow will be $\$1,200 \cdot 1.11 = \$1,332$ at the end of year 2. The following year, the growth rate will be $2\% + (11\% - 2\%)/2 = 6.5\%$, and the cash flow will be \$1,419 at the end of year 3. The growth will be less every year, but it will never reach the inflation rate of 2% perfectly. Next, assume that the appropriate discount rate for a firm this risky is a constant 12%/year. It is not time-varying. (The discount rate on the \$1.2 million cash flow is 12%. The total discount rate for the \$1,332 cash flow in year 2 is thus 25.4%, and so on.) What do you believe the value of this firm to be? (Hint: It is common in pro formas to project forward for a given number of years, say, 5 to 10 years, and then to assume that the firm will be sold for a terminal value, assuming that it has steady growth.)