

## A First Encounter with Capital-Budgeting Rules

### The Internal Rate of Return, and More

This chapter elaborates on the ideas presented in the previous chapter. We still remain in a world of constant interest rates, perfect foresight, and perfect markets. Let's look a little more closely at capital budgeting—the possible decision rules that can tell you whether to accept or reject projects. You already know the answer to the mystery, though: NPV is best. Still, there is one very important alternative to NPV: the internal rate of return, which generalizes the rate of return concept and can often give you good recommendations, too. You will see how these approaches fit together.

One caveat—although you already know the concept of NPV, and although you will learn more about capital-budgeting rules in this chapter, most of the interesting and difficult issues in NPV's application are delayed until Chapter 13 (i.e., after we have covered uncertainty and imperfect markets).

### 4.1 Net Present Value

You have already learned how to use NPV in our perfect world. You first translate cash flows at different points in time into the same units—dollars today—before they can be compared or added. This translation between future values and present values—and its variant, net present value—ranks among the most essential concepts in finance.

But why is NPV the right rule to use? The reason is that, at least in our perfect world with perfect information, a positive-NPV project is the equivalent of free money. For example, if you can borrow or lend money at 8% anywhere today, and you have an investment opportunity that costs \$1 and yields \$1.09, you can immediately contract to receive \$0.01 next year *for free*. (If you wish, discount it back to today, so you can consume it today.) Your rejecting this project would make no sense. Similarly, if you can sell someone an investment opportunity for \$1, which yields only \$1.07 next year, you can again earn \$0.01 *for free*. Again, rejecting this project would make no sense. (Remember that in our perfect world, you can buy or sell projects at will.) Only zero-NPV projects (\$1 cost for \$1.08 payoff) do not allow you to earn free money. Of course, I am using this argument not to show you how to get rich, but to convince you that the NPV rule makes sense and any rule that comes to a different conclusion does not.

Recap: NPV is the most important building block in finance. You must be able to compute it in your sleep.

A "free money" interpretation of NPV.

## IMPORTANT

In a perfect world, if you have all the right inputs to NPV, no other rule can make better decisions. Thus, it is the appropriate decision benchmark—and no other rule can beat it. This also means that information other than the NPV is redundant.

Positive-NPV projects are scarce.

In our perfect world with no uncertainty, logic dictates that positive-NPV projects should be scarce. If they were not scarce and could be found at will, you could get rich too easily. But not just you—everyone with access would want to take on cartloads of them. In real life, the economy would adjust. The “run” on positive-NPV projects would continue until the economy-wide appropriate rate of return (cost of capital) would be bid up to the level where positive-NPV projects are scarce again.

In the real world, NPV is very important, but other measures can provide useful information, too.

As you will find out in later chapters, despite its conceptual simplicity, the application of NPV in the real world is often surprisingly difficult. The primary reason is that you rarely know cash flows and discount factors perfectly. This means that you must estimate them. The secondary reason is that the world is never 100% perfect—that there are absolutely zero taxes, no transaction costs, no disagreements, and infinitely many buyers and sellers. Nevertheless, even in an imperfect market, NPV remains the most important benchmark. Yet other rules may then provide some additional useful information and potentially recommend alternative project choices.

### Separating Investment and Consumption Decisions: Does Project Value Depend on When You Need Cash?

Who owns a project is not important in a perfect capital market.

In our perfect world, when you choose between NPV projects, should you let your preferences about the timing of cash flows influence your decisions? Perhaps you don't want to incur an upfront expense; perhaps you want money today; perhaps you want to defer your consumption and save for the future. Aren't these important factors in making your decision as to which project to choose? The answer is no—the value of any project is its net present value, regardless of your preferences.

The capital markets allow you to shift money across time periods—better than your investment projects can.

In a perfect market, how much cash the owner has also does not matter. Let me explain why. You already know about the time value of money, the fact that cash today is worth more than cash tomorrow. If you do not agree—that is, if you value money tomorrow more than you value money today—then just give it to me until you need it back. I can deposit it in my bank account to earn interest in the interim. In a perfect capital market, you can, of course, do better: You can always shift money between time periods at an “exchange rate” that reflects the time value of money.

Example: Even an “eager” consumer should take the positive-NPV project.

It is this shifting-at-will that explains why ownership does not matter. Assume that you have \$150 cash on hand and that you have exclusive access to a project that costs \$100, and returns \$200 next year. The appropriate interest rate (cost of capital) is 10%—but you *really* want to live it up today. How much can you consume? And, would you take the project? Here is the NPV prescription in a perfect market:

- Sell the project in the competitive market for its NPV:

$$-\$100 + \left( \frac{\$200}{1 + 10\%} \right) = -\$100 + \left( \frac{\$200}{1.10} \right) \approx \$81.82$$

- Spend the  $\$150 + (\$81.82 - \$100) \approx \$231.82$  today. You will be better off taking the project than consuming just your \$150 cash at hand.

Now, assume that you are Austin Powers, the frozen spy, who cannot consume this year. How much will you be able to consume next year? And, would you take the project? NPV tells you what you should do:

- Sell the project in the competitive market for

$$-\$100 + \frac{\$200}{1 + 10\%} \approx \$81.82$$

- Put the \$81.82 into the bank for 10% today. Get \$90 next year.
- Also put your \$150 into the bank at 10% interest to receive \$165 next year.
- Next year, consume \$90 + \$165 = \$255.

Of course, an equally simple solution would be to take the project and just put your remaining \$50 into a bank account.

The point of this argument is simple: Regardless of when you need or want cash (your consumption decision), you are better off taking all positive-NPV projects (your investment decision), and then using the capital markets to shift consumption to when you want it. It makes no sense to let your *consumption decisions* influence your *investment decisions*. This is called the **separation of decisions**: You can make investment decisions without concern for your consumption preferences. (However, this separation of investment and consumption decisions does not always hold in imperfect markets, in which you can face different borrowing and lending interest rates. You might take more projects if you have more cash.)

Here is a simple application of this simplest of insights. After they have lost their clients' money, many brokers like to muddle the truth by claiming that they invested their clients' money for the long term and not for the short term. This excuse presumes that, compared with short-term investments, long-term investments do worse in the short run but better in the long run. However, this makes no sense. See, if your broker had really known that the short-term asset would be better in the short-term, he should have bought it first, realized its higher rate of return over the short-run for you, and then bought you more of the long-term asset (which would now have been relatively cheaper). The fact is that no matter whether an investor needs money sooner or later, a broker should always buy the highest NPV investments. In the end, this is what is best for all clients.

### Errors: Mistakes in Cash Flow versus Cost of Capital Estimates

Although it would be better to get everything perfect, it is often impossible to come up with perfect cash flow forecasts and appropriate interest rate estimates. Everyone makes errors when outcomes in the world are uncertain. How bad are estimation mistakes? Is it worse to commit an error in estimating cash flows or in estimating the cost of capital? To answer these questions, we will do a simple form of **scenario analysis**, in which we consider a very simple project to learn how changes in our estimates matter to the ultimate present value. Scenario analysis is also essential for managers, who need to learn how sensitive their estimated value is to reasonable alternative possible outcomes. Therefore, this method is also called a **sensitivity analysis**. (It becomes even more important when you work with real options in Chapter 13.)

**Short-term projects:** Assume that your project will pay off \$200 next year, and the proper interest rate for such projects is 8%. Thus, the correct project present value is

$$\text{Correct PV} = \frac{\$200}{1 + 8\%} \approx \$185.19$$

A "sleeper" consumer should also take the positive-NPV project.

The moral of the story: Consumption and investment decisions can be separated in a perfect capital market.

➤ [Imperfect markets, lack of separation,](#)  
Sect. 11.1, Pg.251.

Investing for the long-run is the same as investing for the short-run.

In the real world, it is often impossible to get the NPV inputs perfectly correct.

The benchmark case: A short-term project, correctly valued.

Committing an error in cash flow estimation.

If you make a 10% error in your cash flow, mistakenly believing it to return \$220, you will compute the present value to be

$$\text{Cash Flow Error PV} = \frac{\$220}{1 + 8\%} \approx \$203.70$$

The difference between \$203.70 and \$185.19 is a 10% error in your present value.

Committing an error in interest rate estimation.

In contrast, if you make a 10% error in your cost of capital (interest rate), mistakenly believing it to require a cost of capital (expected interest rate) of 8.8% rather than 8%, you will compute the present value to be

$$\text{Discount Rate Error PV} = \frac{\$200}{1 + 8.8\%} \approx \$183.82$$

The difference between \$183.82 and \$185.19 is less than \$2, which is an error of about 1%. In sum, discount rate errors tend to be less harmful than cash flow errors for short-run projects.

A long-term project, correctly valued and incorrectly valued.

**Long-term projects:** Now take the same example but assume the cash flow will occur in 30 years. The correct present value is now

$$\text{Correct PV} = \frac{\$200}{(1 + 8\%)^{30}} = \frac{\$200}{1.08^{30}} \approx \$19.88$$

The 10% “cash flow error” present value is

$$\text{Cash Flow Error PV} = \frac{\$220}{(1 + 8\%)^{30}} = \frac{\$220}{1.08^{30}} \approx \$21.86$$

and the 10% “interest rate error” present value is

$$\text{Discount Rate Error PV} = \frac{\$200}{(1 + 8.8\%)^{30}} = \frac{\$200}{(1.088\%)^{30}} \approx \$15.93$$

Both cash flow errors and cost of capital errors are important for long-term projects.

This calculation shows that cash flow estimation errors and interest rate estimation errors are now both important. For longer-term projects, estimating the correct interest rate becomes relatively more important. Yet, though correct, this argument may be misleading. Estimating cash flows 30 years into the future often seems more like voodoo than science. Your uncertainty usually explodes over longer horizons. In contrast, your uncertainty about the long-term cost of capital tends to grow very little with the time horizon—you might even be able to ask your investors today what they demand as an appropriate cost of capital for a 30-year investment! Of course, as difficult as cash flow estimation may be, you have no alternative. You simply must try to do your best at forecasting.

## IMPORTANT

- For short-term projects, errors in estimating correct interest rates are less problematic in computing NPV than are errors in estimating future cash flows.
- For long-term projects, errors in estimating correct interest rates and errors in estimating future cash flows are both problematic in computing NPV. Nevertheless, in reality, you will tend to find it more difficult to estimate far-away future cash flows (and thus you will face more errors) than to estimate the appropriate discount rate demanded by investors today for far-away cash flows.

**Q 4.1.** What is the main assumption that allows you to consider investment (project) choices without regard to when you need wealth (or how much money you currently have at hand)?

**Q 4.2.** You have \$500 and really, really want to go to the Superbowl tonight (which would consume all your cash). You cannot wait until your project completes: This project would cost \$400 and offer a rate of return of 15%, although equivalent interest rates are only 10%. If the market is perfect, what should you do?

## 4.2 The Internal Rate of Return (IRR)

There is another common capital-budgeting method, which often leads to the same recommendations as the NPV rule. This method is useful because it does so through a different route and often provides good intuition about the project.

Let's assume that you have a project with cash flows that translate into a rate of return of 20% (e.g., \$100 investment, \$120 payoff), and the prevailing discount rate is 10%. Because your project's rate of return of 20% is greater than the prevailing discount rate of 10%, you should intuitively realize that it is a good one. It is also a positive-NPV project—in the example,  $-\$100 + \$120/1.1 \approx \$9.10$ .

There is only one problem: How would you compute the rate of return on a project or bond that has many different payments? For example, say the investment costs \$100,000 and pays off \$5,000 in one year, \$10,000 in two years, and \$120,000 in three years. What is the rate of return of this project? Think about it. The rate of return formula works only if you have exactly one inflow and one outflow. This is not the case here. What you need now is a "kind of rate of return" (a "statistic") that can take many inflows and outflows and provide something similar to a rate of return. If there is only one of each, it should give the same number as the simple rate of return.

Such a measure exists. It is called the internal rate of return (IRR). The word "internal" is an indicator that the rate is intrinsic to your project, depending only on its cash flows.

IRR  $\approx$  NPV.

Our new capital-budgeting method compares the project's rate of return to the prevailing rate of return.

We need a "sort-of average rate of return" that is implicit in future cash flows.

The IRR is this characteristic that describes multiple cash flows.

### IMPORTANT

- The **internal rate of return (IRR)** is the quantity, which, given a complete set of cash flows, solves the NPV formula set to zero,

$$0 = C_0 + \frac{C_1}{1 + \text{IRR}} + \frac{C_2}{(1 + \text{IRR})^2} + \frac{C_3}{(1 + \text{IRR})^3} + \dots \quad (4.1)$$

- If there are only two cash flows, the IRR is the rate of return. Thus, the IRR generalizes the concept of rate of return to multiple cash flows. Every rate of return is an IRR, but the reverse is not the case.
- The IRR itself is best thought of as a characteristic of project cash flows.

The internal rate of return is such a common statistic in the context of bonds that it has acquired a second name: the **yield-to-maturity (YTM)**. There is no difference between the IRR and the YTM.

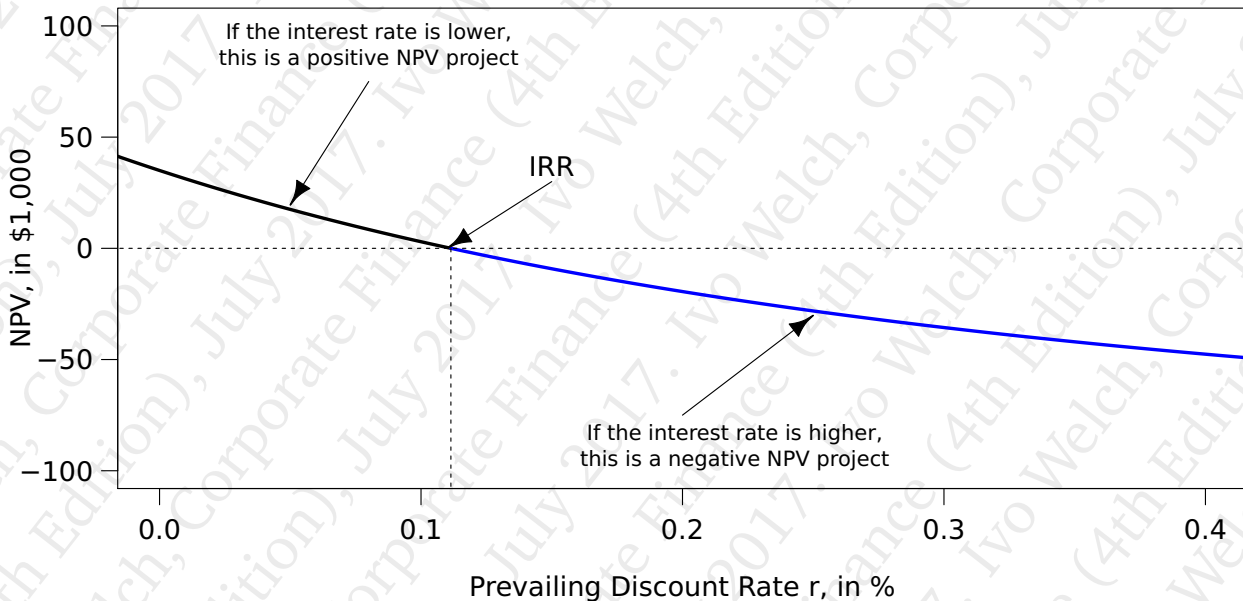
YTM is the same as IRR.

Let's illustrate the IRR. First, if there is only one inflow and one outflow, the IRR is the simple rate of return. For example, if a simple project costs \$100 today and pays \$130 next year, the IRR is obtained by solving

IRR generalizes rates of return: A simple project's rate of return is its IRR.

$$-\$100 + \frac{\$130}{1 + \text{IRR}} = 0 \Leftrightarrow \text{IRR} = \frac{\$130 - \$100}{\$100} = 30\%$$

$$C_0 + \frac{C_1}{1 + \text{IRR}} = 0 \Leftrightarrow \text{IRR} = \frac{C_1 - C_0}{C_0}$$



**Exhibit 4.1:** NPV as a Function of the Interest Rate. This figure draws the NPV for a project that costs \$100,000 and pays \$5,000, \$10,000, and \$120,000 in consecutive years. The IRR is the x-coordinate where the NPV function intersects the zero-line.

Here is an iteration method that shows how you can solve the IRR equation yourself.

Now consider an example where a simple rate of return won't work: What number would best characterize the implied rate of return for a project that costs \$100,000 today and that will yield \$5,000, \$10,000, and \$120,000? You cannot compute a simple rate of return with four cash flows. Exhibit 4.1 shows you the NPV of this project as a function of the prevailing interest rate. If the discount rate is very low, then the NPV is positive. IRR is the interest rate that makes the NPV exactly equal to zero. In this case, this means that you should solve

$$0 = -\$100,000 + \frac{\$5,000}{1 + \text{IRR}} + \frac{\$10,000}{(1 + \text{IRR})^2} + \frac{\$120,000}{(1 + \text{IRR})^3}$$

$$0 = C_0 + \frac{C_1}{1 + \text{IRR}} + \frac{C_2}{(1 + \text{IRR})^2} + \frac{C_3}{(1 + \text{IRR})^3}$$

What is the discount rate that sets the NPV equation to zero? If you do not want to draw the full figure to find out where your NPV function crosses the zero axis, then you can try to solve such equations by trial and error. Start with two values, say, 5% and 10%.

$$-\$100,000 + \frac{\$5,000}{1 + 10\%} + \frac{\$10,000}{(1 + 10\%)^2} + \frac{\$120,000}{(1 + 10\%)^3} \approx \$2,968$$

To reach zero, you need to slide above 10%. Try 11% and 12%,

$$-\$100,000 + \frac{\$5,000}{1 + 11\%} + \frac{\$10,000}{(1 + 11\%)^2} + \frac{\$120,000}{(1 + 11\%)^3} \approx \$364$$

$$-\$100,000 + \frac{\$5,000}{1 + 12\%} + \frac{\$10,000}{(1 + 12\%)^2} + \frac{\$120,000}{(1 + 12\%)^3} \approx -\$2,150$$

Okay, the solution is closer to 11%. A lucky trial reveals

$$-\$100,000 + \frac{\$5,000}{1 + 11.14252\%} + \frac{\$10,000}{(1 + 11.14252\%)^2} + \frac{\$120,000}{(1 + 11.14252\%)^3} \approx 0$$

Therefore, the answer is that this project has an IRR of about 11.14%. You can think of the internal rate of return as a sort-of average rate of return embedded in the project's cash flows.

There is no easy general formula to compute the IRR if you are dealing with more than three cash flows. However, an automated function to compute an IRR is built into modern computer spreadsheets and usually precludes the need to solve algebraic equations by trial-and-error. Exhibit 4.2 (row 1) shows how you would find the IRR for this project in a spreadsheet.

Spreadsheets make it easy to find the IRR fast.

	A	B	C	D	E	
1	-100,000	5,000	10,000	120,000	=IRR(A1:D1)	← E1 will become 11.142%
2	100,000	-5,000	-10,000	-120,000	=IRR(A2:D2)	← E2 will become 11.142%
3	-1,000	600	600		=IRR(A3:C3)	← D3 will become 13%

**Exhibit 4.2:** IRR Calculations in a Computer Spreadsheet (Excel or OpenOffice). The first line is the project worked out in the text. The second line shows that the negative of the project has the same IRR. The third line is just another example that you can check for yourself.

Note that the negative cash flow pattern in row 2 of Exhibit 4.2 has the same IRR. That is, receiving an inflow of \$100,000 followed by payments of \$5,000, \$10,000, and \$120,000 also has an 11.14252% internal rate of return. You can see that this must be the case if you look back at the IRR formula. Any multiplicative factor (like -1) simply cancels out and therefore has no impact on the solution.

Multiplying all cash flows by the same factor does not change the IRR.

$$\begin{aligned} 0 &= \text{Factor} \cdot C_0 + \frac{\text{Factor} \cdot C_1}{1 + \text{IRR}} + \frac{\text{Factor} \cdot C_2}{(1 + \text{IRR})^2} + \frac{\text{Factor} \cdot C_3}{(1 + \text{IRR})^3} + \dots \\ &= \text{Factor} \cdot \left[ C_0 + \frac{C_1}{1 + \text{IRR}} + \frac{C_2}{(1 + \text{IRR})^2} + \frac{C_3}{(1 + \text{IRR})^3} + \dots \right] \\ &= C_0 + \frac{C_1}{1 + \text{IRR}} + \frac{C_2}{(1 + \text{IRR})^2} + \frac{C_3}{(1 + \text{IRR})^3} + \dots \end{aligned}$$

**Q 4.3.** From memory, write down the equation that defines IRR.

**Q 4.4.** What is the IRR of a project that costs \$1,000 now and produces \$1,000 next year?

**Q 4.5.** What is the IRR of a project that costs \$1,000 now and produces \$500 next year and \$500 the year after?

**Q 4.6.** What is the IRR of a project that costs \$1,000 now and produces \$600 next year and \$600 the year after?

**Q 4.7.** What is the IRR of a project that costs \$1,000 now and produces \$900 next year and \$900 the year after?

**Q 4.8.** A project has cash flows of  $-\$100$ ,  $\$55$ , and  $\$70$  in consecutive years. Use a spreadsheet to find the IRR.

**Q 4.9.** What is the YTM of an  $x\%$  annual level-coupon bond whose price is equal to the principal paid at maturity? For example, take a 5-year bond that costs \$1,000 today, pays 5% coupon ( $\$50$  per year) for 4 years, and finally repays \$1,050 in principal and interest in year 5.

**Q 4.10.** What is the YTM of a 5-year zero-bond that costs \$1,000 today and promises to pay \$1,611?

**Q 4.11.** Compute the yield-to-maturity of a two-year bond that costs \$25,000 today and pays \$1,000 at the end of each of the 2 years. At the end of the second year, it also repays \$25,000. What is the bond's YTM?

### Projects with Multiple or No IRRs

When projects have many positive and many negative cash flows, they can often have multiple internal rates of return. For example, take a project that costs \$100,000, pays \$205,000, and has environmental cleanup costs of \$102,000. Exhibit 4.3 shows that this project has two internal rates of return:  $r = -15\%$  and  $r = 20\%$ . Confirm this:

$$\begin{aligned}
 -\$100,000 + \frac{\$205,000}{1 + (-15\%)} + \frac{-\$102,000}{[1 + (-15\%)]^2} &= 0 \\
 -\$100,000 + \frac{\$205,000}{1 + 20\%} + \frac{-\$102,000}{(1 + 20\%)^2} &= 0
 \end{aligned}$$

Huh? So does this project have an internal rate of return of  $-15\%$  or an internal rate of return of  $20\%$ ? The answer is both—the fact is that both IRRs are valid according to the definition. And don't think the number of possible solutions is limited to two—with other cash flows, there could be dozens. What do computer spreadsheets do if there are multiple IRRs? You may never know. They usually just pick one for you. They don't even give you a warning.

While some projects have multiple IRRs, other projects have none. For example, what is the internal rate of return of a project that yields \$10 today and \$20 tomorrow (that is, it never demands an investment)? Such a project has no internal rate of return. The NPV formula is never zero, regardless of what the prevailing interest rate is. This makes sense, and the fact that there is no IRR is pretty obvious from the cash flows. After all, they both have the same sign. But what is the IRR of a project that has a cost of \$10,000, then pays \$27,000, and finally requires a cleanup cost of \$20,000? Exhibit 4.3 shows that such a project also has no rate of return at which its NPV would turn positive. Therefore, it has no IRR. What do computer spreadsheets do if there are no IRRs? Thankfully, most of the time, they give an error message that will alert you to the problem.

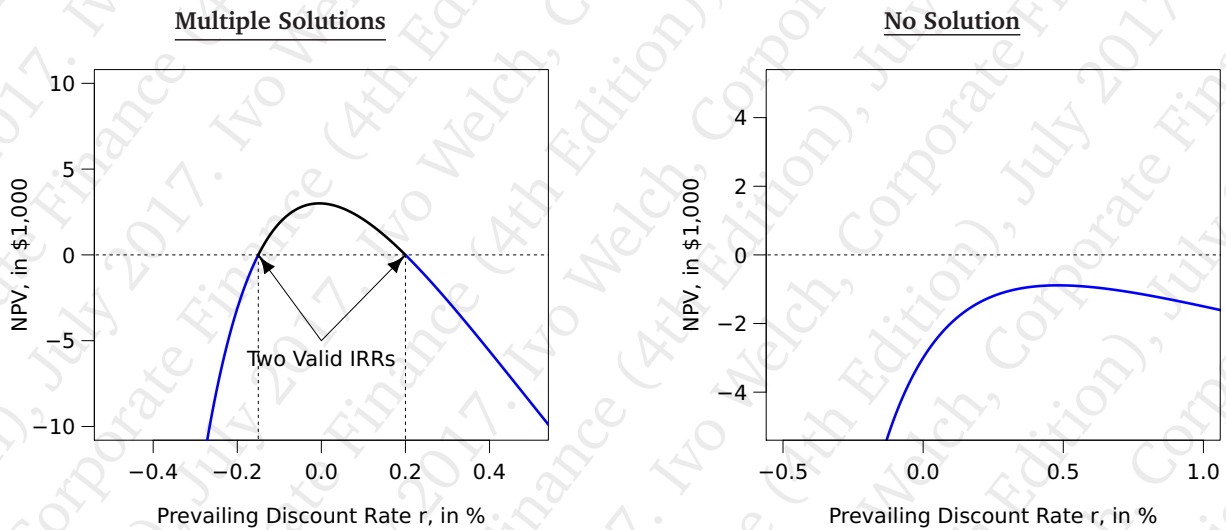
Can you ever be sure that your project has one unique internal rate of return? Yes. It turns out that if you have one negative cash flow followed only by positive cash flows—which happens to be far and away the most common investment pattern—then your project has one and only one IRR. (Projects with cash flows with many different positive and negative signs can still have only one IRR, but it's not guaranteed.) Partly because bonds have such cash flow patterns, YTM is even more popular than IRR. Obviously, you also have a unique IRR if a project has the opposite cash flow pattern—that is, a positive cash inflow followed only by negative cash flows.

Here is an example of a project with two IRRs.

Projects that have all negative or all positive cash flows have no IRRs—but so do some other projects.

The most common types of investment projects have a unique IRR, because they have one outflow followed only by inflows (or vice-versa).





**Exhibit 4.3:** Multiple and No IRR Solutions. The left figure draws the NPV for a project that costs \$100,000, pays \$205,000, and then has cleanup costs of \$102,000. The right figure draws the NPV for a project that costs \$10,000, pays \$27,000, and then requires a \$20,000 cleanup cost.

Q 4.12. Give an example of a problem that has multiple IRR solutions.

Q 4.13. Give an example of a project that has no IRR.

Q 4.14. For the following projects A through G, plot the NPVs as a function of the prevailing interest rate and determine the appropriate IRRs.

	Y0	Y1	Y2	Y3	Y4
A	+\$1,000	-\$5,000	+\$9,350	-\$7,750	+\$2,402.4
B	+\$50,000	-\$250,000	+\$467,500	-\$387,500	\$120,120
C	+\$100,000	-\$250,000	+\$200,000		
D	-\$100	+\$300	-\$400	+\$400	
E	+\$100	-\$300	+\$400	-\$400	
F	+\$200	-\$600	+\$800	-\$800	
G	-\$100	+\$300	-\$200		

## IRR as a Capital-Budgeting Rule

IRR rule often yields the same result as the NPV rule.

One important reason why IRR is so useful is that it can often substitute for NPV as an investment criterion.

### IMPORTANT

- The IRR capital-budgeting rule states that if and only if an investment project's IRR (a characteristic of project cash flows) is above the appropriate discount rate (i.e., the cost of capital quoted like a required interest rate) for the project, then the project should be taken. In this context, the cost of capital is often called the **hurdle rate**.

In many cases, the IRR capital-budgeting rule gives the same correct answer as the NPV capital-budgeting rule. However, there are some delicate situations in which this is not the case. This will be explained below.

Confirm that the IRR and NPV capital-budgeting rules give the same recommendation.

Let me illustrate that you usually get the same answer. Return to our project that costs \$100,000 and yields \$5,000, \$10,000, and \$120,000 with its IRR of 11.14%. The IRR capital-budgeting rule states that if the prevailing cost of capital in the economy (i.e., the hurdle rate) to finance our project is 11.20%, then you should not take this project. If it is 11.10%, then you should take this project. Does NPV offer the same recommendation? Try it:

$$\text{NPV at 11.10\%} = -\$100,000 + \frac{\$5,000}{1 + 11.10\%} + \frac{\$10,000}{(1 + 11.10\%)^2} + \frac{\$120,000}{(1 + 11.10\%)^3} \approx +\$108$$

$$\text{NPV at 11.20\%} = -\$100,000 + \frac{\$5,000}{1 + 11.20\%} + \frac{\$10,000}{(1 + 11.20\%)^2} + \frac{\$120,000}{(1 + 11.20\%)^3} \approx -\$146$$

Indeed, you get the same recommendation.

If the cash flow is negative, the IRR stays the same, but the take-it-or-leave-it rule reverses.

If the cash flows are the exact opposite—that is, if you receive \$100,000 upfront and pay out \$5,000, \$10,000, and \$120,000—then this would not really be an investment project, but more like investment financing. You would now want to take this financing alternative if and only if the prevailing interest rate is *above* 11.14%. Be careful about whether you want your IRR to be above or below the hurdle rate! (My advice to avoid such errors is to always work out the NPV, too—it will never mislead you.)

IRR can be computed before the cost of capital is known.

Why use the IRR instead of the NPV investment criterion? The answer is that the former is often quite intuitive and convenient, provided that the project's cash flow stream implies one unique IRR. In this case, IRR is convenient because you can compute it without having looked at financial markets, interest rates, or costs of capital. This is IRR's most important advantage over NPV: *It can be calculated even before you know the appropriate interest rate (cost of capital).* Moreover, IRR can give you useful project information in and of itself. It is also helpful in judging project profitability and thereby allows you to judge the performance of a manager—it is often easier to hold her to her earlier promise of delivering an IRR of 20% than it is to argue with her about what the appropriate cost of capital for her project should be. And, finally, by comparing the IRR to the cost of capital, you can determine how much “buffer” you have in terms of getting your cash flow estimates wrong by a certain percentage and still be correct in your ultimate decision as to whether to take the project or not.

IRR is a characteristic of a project's cash flows. (It is not an interest rate.)

**Q 4.15.** A project has cash flows of  $-\$1,000$ ,  $-\$2,000$ ,  $+\$3,000$ , and  $+\$4,000$  in consecutive years. Your cost of capital is 30% per annum. Use the IRR rule to determine whether you should take this project. Does the NPV rule recommend the same action?

**Q 4.16.** A project has cash flows of  $-\$1,000$ ,  $-\$2,000$ ,  $-\$3,000$ ,  $+\$4,000$ , and  $+\$5,000$  in consecutive years. Your cost of capital is 20% per annum. Use the IRR rule to determine whether you should take this project. Confirm your recommendation using the NPV rule.

**Q 4.17.** A project has cash flows of +\$200, −\$180, −\$40 in consecutive years. The prevailing interest rate is 5%. Should you take this project?

**Q 4.18.** You can invest in a project with diminishing returns. Specifically, the formula relating next year's payoff to your investment today is  $C_1 = \sqrt{-C_0}$ , where  $C_0$  and  $C_1$  are measured in millions of dollars. For example, if you invest \$100,000 in the project today, it will return  $\sqrt{\$0.1} \approx \$0.316$  million next year. The prevailing interest rate is 5% per annum. Use a spreadsheet to answer the following two questions:

1. What is the IRR-maximizing investment choice? What is the NPV at this choice?
2. What is the NPV-maximizing investment choice? What is the IRR at this choice?

### Problems with IRR as a Capital-Budgeting Rule

If you use IRR *correctly* and in the right circumstances, it can give you the same answer as the NPV rule. You cannot do better than doing it correctly, so it is always safer to use the NPV rule than the IRR rule. When does the IRR capital-budgeting rule work well? If there is only one unique IRR, it is often an elegant method. Of course, as just stated, you still have to make sure that you get the sign right. If your project requires an upfront outflow followed by inflows, you want to take the project if its IRR is *above* your cost of capital. If the project is financing (like debt, which has an upfront inflow followed by outflows), you want to take this project if its IRR is *below* your cost of capital. My advice is to use NPV as a check of your IRR calculations in any case.

Unfortunately, if the IRR is not unique (and recall that there are projects with multiple IRRs or no IRR), then the IRR criterion becomes outright painful. For example, if your prevailing cost of capital is 9% and your project has IRRs of 6%, 8%, and 10%, should you take this project or avoid it? The answer is not obvious. In this case, to make an investment decision, you are better off falling back to drawing a part of the NPV graph in one form or another. My advice: just avoid IRR. (Yes, it is possible to figure out how to use IRR, depending on whether the NPV function crosses the 0-axis from above or below, but working with IRR under such circumstances only begs for trouble, i.e., mistakes. There is also a “modified IRR” [the so-called MIRR] measure that can sometimes eliminate multiple solutions. (MIRR is not worth the trouble.) If you have a project without any valid IRR, you again have to fall back to NPV, but using NPV will be simpler. Just work out whether the NPV function is above or below the 0-axis for any arbitrary discount rate (e.g.,  $r = 0$ ), and use this to decide whether to take or reject your project.

There are two more problems when using IRR that you need to be aware of:

**1. Project comparisons and scale:** The IRR criterion can be misleading when projects are mutually exclusive. For example, if you had to choose, would you always prefer a project with a 100% IRR to a project with a 10% IRR? Think about it.

What if the first project is an investment opportunity of \$5 (returning \$10), and the second project is an investment opportunity of \$1,000 (returning \$1,100)? Take the case where the prevailing discount rate is 5% per annum. Then,

	Y0	Y1	IRR	NPV at 5%
A	−\$5	+\$10	100%	+\$4.52
B	−\$1,000	+\$1,100	10%	+\$47.62

If you can only take one project, then you should take project B, even though its IRR is much lower than that of project A.

IRR is safe to use when there is only one positive or only one negative cash flow.

IRR often fails in nonobvious ways when there are multiple negative or positive cash flows.

Two more problems: (1) IRR has no concept of scale; (2) there may not be an obvious hurdle rate to compare it to.

**2. Cost of capital comparison:** The next chapter will explain that long-term interest rates are often higher than short-term interest rates. For example, in mid-2016, a 1-year Treasury bond offered a rate of return of 0.5%, while a 30-year Treasury bond offered an annualized rate of return of 2.5%. Let's assume that your project is risk-free, too. Should you take a risk-free project that has an IRR of 1.5%? There is no clear answer.

These two problems may seem obvious when highlighted in isolation. But in the context of complex, real-world, multiple-project analyses, they are surprisingly often overlooked.

**Q 4.19.** What are the problems with the IRR computation and criterion?

**Q 4.20.** The prevailing interest rate is 25%. If the following two projects are mutually exclusive, which should you take?

	Y0	Y1	Y2	Y3	Y4
A	+\$50,000	-\$250,000	+\$467,500	-\$387,500	+\$120,120
B	-\$50,000	+\$250,000	-\$467,500	+\$387,500	-\$120,120

What does the NPV rule recommend? What does the IRR rule recommend?

**Q 4.21.** The prevailing interest rate is 25%. If the following two projects are mutually exclusive, which should you take?

	Y0	Y1	Y2	Y3
A	+\$500,000	-\$200,000	-\$200,000	-\$200,000
B	+\$50,000	+\$25,000		

What does the NPV rule recommend? What does the IRR rule recommend?

**Q 4.22.** The prevailing interest rate is 10%. If the following three projects are mutually exclusive, which should you take?

	Y0	Y1	Y2
A	-\$500	+\$300	+\$300
B	-\$50	+\$30	+\$30
C	-\$50	+\$35	+\$35

What does the NPV rule recommend? What does the IRR rule recommend?

**Q 4.23.** The prevailing interest rate is 5% over the first year and 10% over the second year. That is, over two years, your compounded interest rate is  $(1 + 5\%) \cdot (1 + 10\%) - 1 = 15.5\%$ . Your project costs \$1,000 and will pay \$600 in the first year and \$500 in the second year. What does the IRR rule recommend? What does the NPV rule recommend?

### 4.3 The Profitability Index

A less prominent measure sometimes used in capital budgeting is the **profitability index**. It divides the present value of future cash flows by the project cost (the negative of the first cash flow). For example, if you have a project with cash flows

How the probability index is computed.

	Y0	Y1	Y2	Y3	PV(Y1 to Y3)
Project A Cash Flow	-\$100	\$70	\$60	\$50	\$128.94

and the interest rate is 20% per annum, you would first compute the present value of future cash flows as

$$\begin{aligned} PV &= \frac{\$70}{1.2} + \frac{\$60}{1.2^2} + \frac{\$50}{1.2^3} \approx \$128.94 \\ &= PV(C_1) + PV(C_2) + PV(C_3) \end{aligned}$$

Subtract the \$100 upfront cost, and the NPV is \$28.94. The profitability index is

$$\begin{aligned} \text{Profitability Index} &= \frac{\$128.94}{-(-\$100)} \approx 1.29 \\ \text{Profitability Index} &= \frac{PV(\text{Future Cash Flows})}{\text{Original Cost}} \end{aligned}$$

A positive-NPV project usually has a profitability index above 1—“usually” because the profitability index is meaningful only if the first cash flow is a cash outflow. When this is the case, you can use either NPV or the profitability index for a simple “accept/reject” decision: The statements “NPV > 0” and “profitability index > 1” are the same. That is, like IRR, the profitability index can give the correct answer in the most common situation of one negative cash flow upfront followed by all positive cash flows thereafter.

A profitability index-based capital-budgeting rule can give the same answer as IRR (and NPV).

Some managers like the fact that the profitability index gives information about relative performance and use of capital. For example,

Here it works nicely, and may even convey some information above and beyond IRR.

	Y0	Y1	Y2	Y3	PV(Y1 to Y3)
Project B Cash Flow	-\$10.00	\$21.14	\$18.12	\$15.10	\$38.94

has the same NPV of \$28.94 as the original project, but B’s profitability index is higher than 1.29 because it requires less capital upfront.

$$\begin{aligned} \text{Profitability Index} &= \frac{\$38.94}{-(-\$10)} \approx 3.89 \\ \text{Profitability Index} &= \frac{PV(\text{Future Cash Flows})}{\text{Original Cost}} \end{aligned}$$

The reason is that the profitability index values the scale of the project differently. It is intuitively apparent that you would prefer the second project, even though it has the same NPV, because it requires less capital. It may even be less risky, but this can be deceiving, because we have not specified the risk of the future cash flows.

Unfortunately, this feature that you just considered as an advantage can also be a disadvantage. You cannot use the profitability index to choose among different projects. For example, assume that your first project returns twice as much in cash flow in all future periods, so it is clearly the better project now.

But here is where the profitability index can go wrong: Like IRR, it has no concept of scale.

	Y0	Y1	Y2	Y3	PV(Y1 to Y3)	NPV	Profitability Index
B	-\$10	\$21.14	\$18.12	\$15.10	\$38.94	\$28.94	$\frac{\$38.94}{-(-\$10)} \approx 3.89$
C	-\$100	\$140	\$120	\$100	\$257.87	\$157.87	$\frac{\$257.87}{-(-\$100)} \approx 2.58$

Note that the profitability index of project C is less than that of project B. The reason is that, when compared to NPV, the profitability index *really* “likes” lower-upfront investment projects. It can therefore indicate higher index values even when the NPV is lower. This is really the same scale problem that popped up when we tried to use IRR for comparing mutually exclusive projects. Both look at relative “percentage” performance, not at the dollar gain, like NPV does. You should really consider the profitability index in choosing among projects only if the NPVs of the two projects are equal (or at least very similar).

**Q 4.24.** The prevailing interest rate is 10%. If the following three projects are mutually exclusive, which should you take?

	Y0	Y1	Y2
A	-\$500	+\$300	+\$300
B	-\$50	+\$30	+\$30
C	-\$50	+\$35	+\$35

You have already worked out the recommendations of the NPV and the IRR rule. What does the profitability rule recommend?

#### 4.4 The Payback Capital-Budgeting Rule

The most common aberrant capital-budgeting rule in the real world is the payback rule.

Three sample projects.

What if you want something more “practical” than the eggheaded “theoretical” capital-budgeting methods? Aren’t there easier methods that can help you make investment decisions? Yes, they exist—and they usually result in bad practical choices. Indeed, after IRR and NPV, the most commonly used capital-budgeting rule is a “practical” one, the **payback rule**. You need to know why you should not fall for it.

Under the payback rule, projects are assumed to be better if you can recover their original investment faster. For the most part, this is a stupid idea. Consider the following three projects:

	Y1	Y2	Y3	Y4	Payback Period
A	-\$5	+\$8			1 year
B	-\$5	+\$4	\$100		2 years
C	-\$5	+\$4	\$0	\$100,000	3 years

Here is why choosing projects based solely on payback speed is dumb.

Project A has the shortest (best) payback period, but it is the worst of the three projects (assuming common discounting rates). Project B has the next shortest payback period, but it is the second-worst of the three projects (assuming reasonable interest rates). Project C has the longest (worst) payback period, but is the best project. There is also a version of payback in which future paybacks are discounted (**discounted payback**). This measure asks not how long it takes to get your money back, but how long it takes to get the *present value* of your money back. It is still a bad idea.

To be fair, payback can be an interesting number.

1. There is a beautiful simplicity to payback. Everyone will understand “you will get your money back within five years,” but not everyone will understand “the NPV is \$50 million.”
2. Payback’s emphasis on earlier cash flows helps firms set criteria when they don’t trust their managers. For instance, if your department manager claims that you will get your money back within one year, and three years have already passed without your having seen a penny, then something is probably wrong and you may need a better manager.
3. Payback can also help if you are an entrepreneur with limited capital, faced with an imperfect capital market. In such cases, your cost of capital can be very high and getting your money back in a short amount of time is paramount. The payback information can help you assess your future “liquidity.”
4. Finally, in many ordinary situations, in which the choice is a pretty clear-cut yes or no, the results of the payback rule may not lead to severe mistakes (as would a rule that would ignore all time value of money). If you have a project in which you get your money back within one month, chances are that it’s not a bad one, even from an NPV perspective. If you have a project in which it takes fifty years to get your money back, chances are that it has a negative NPV.






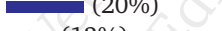
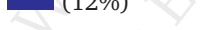
Having said all this, if you use payback to make decisions, it can easily lead you to take the wrong projects and ruin your company. Why take a chance when you know better capital-budgeting methods? My view is that it is not a bad idea to work out the payback period and use it as “interesting supplemental information,” but you should never base project choices on it—and you should certainly never compare different projects primarily on the basis of payback.

In fairness, the speed of payback can be an interesting statistic.

► [Entrepreneurial finance](#), Sect. 11.5, Pg.269.

It is best to avoid payback as a primary decision rule.

#### 4.5 How Do Executives Decide?

Method	CFO Usage	Yields Correct Answer	Main Explanation
Internal Rate of Return (IRR)	 (76%)	Often	Chapter 4
Net Present Value (NPV)	 (75%)	(Almost) Always	Chapter 2
Payback Period	 (57%)	Rarely	Chapter 4
Earning Multiples (P/E Ratios)	 (39%)	With Caution	Chapter 15
Discounted Payback	 (30%)	Rarely	Chapter 4
Accounting Rate of Return	 (20%)	Rarely	Chapter 15
Profitability Index	 (12%)	Often	Chapter 4

**Exhibit 4.4:** *CFO Valuation Techniques*. Rarely means “usually no—often used incorrectly in the real world.” NPV works if correctly applied, which is why I added the qualifier “almost” to always. Of course, if you are considering an extremely good or an extremely bad project, almost any evaluation criterion is likely to give you the same recommendation. (Even a stopped clock gives you the correct time twice a day.) Source: John Graham and Campbell Harvey, 2001.

A survey asked CFOs what they use. It found the good methods (NPV and IRR) are most important.

The two unexplained methods (P/E and accounting rate of return) in the table are based on accounting numbers.

► [Financials](#), Chapter 14, Pg.363.

► [Comparables](#), Chapter 15, Pg.395.

► [ROE=Accounting rate of return](#), Formula 15.6, Pg.424.

► [Warning about BV stock numbers](#), Sect. 14.7, Pg.388.

The survey unfortunately did not ask managers whether they select projects primarily to increase earnings—a pity.

Accounting-based rules are problematic.

In real-life, the best projects are often not taken. Math may just be a rhetorical weapon to win the political fights to convince others to fund projects.

So what do managers really use for capital budgeting? In their 2001 survey (and regular updates thereafter), Graham and Harvey (from Duke University) asked 392 managers, primarily **chief financial officers (CFOs)**, what techniques they use when deciding on projects or acquisitions. The results are listed in Exhibit 4.4. The two most prominent measures are also the correct ones: They are the “internal rate of return” and the “net present value” methods. Alas, the troublesome “payback period” method and its cousin, the “discounted payback period,” still remained surprisingly common. An updated 2016 paper by Mukhlynina and Nyborg finds that valuation practitioners nowadays usually use *both* multiples and discounted cash flow analysis, with frequencies in the mid-80s.

Of course, this is your first encounter with capital-budgeting rules, and there will be a lot more details and complications to come (especially for NPV). Let me also briefly explain the two methods mentioned in the table that you do not know yet: the “earnings multiples” and the “accounting rate of return” methods. They will be explained in great detail in Chapters 14 and 15. In a nutshell, the “earnings multiples” method tries to compare your project’s earnings directly to the earnings of other firms in the market. If your project costs less and earns more than these alternative opportunities, then the multiples approach usually suggests you take it. It can often be useful, but considerable caution is warranted. The “accounting rate of return” method uses an accounting “net income” and divides it by the “book value of equity.” This is rarely a good idea—financial accounting is not designed to accurately reflect firm value. (Accounting statements are relatively better in measuring flows [like earnings] than they are in measuring stocks [like book value].)

Graham and Harvey did not allow respondents to select a third measure for project choice: a desire to maximize reported earnings. Managers care about earnings, especially in the short run and just before they are up for a performance evaluation or retirement. Thus, they may sometimes pass up good projects for which the payoff is far in the future.

As you will learn, rules that are based on accounting conventions and not on economics are generally not advisable. I almost always recommend against using them. I have no idea what kind of projects you will end up with if you were to follow their recommendations—except that in many cases, if the measures are huge (e.g., if the accounting rate of return is 190% per annum), then chances are that the project has positive NPV, too.

One view, perhaps cynical, is that all the capital-budgeting methods you have now learned give you not only the tools to choose the best projects but also the language to argue intelligently and professionally to get your favorite projects funded. In many corporations, “power” rules. The most influential managers get disproportionately large funding for their projects. This is of course not a good objective, much less a quantitative value-maximization method for choosing projects.

## Summary

This chapter covered the following major points:

- If the market is perfect and you have the correct inputs, then net present value is the undisputed correct method to use.
- In a perfect market, projects are worth their net present values. This value does not depend on who the owner is or when the owner needs cash. Any owner can always take the highest NPV projects and use the capital markets to shift cash into periods in

which it is needed. Therefore, consumption and investment decisions can be made independently.

- The internal rate of return, IRR, is computed from a project’s cash flows by setting the NPV formula equal to zero.
- The internal rate of return does not depend on the prevailing cost of capital. It is a project-specific measure. It can be interpreted as a “sort-of-average” rate of return implicit in many project cash flows. Unlike



a simple rate of return, it can be computed when a project has more than one inflow and outflow.

- Projects can have multiple IRR solutions or no IRR solutions.
- Investment projects with IRRs above their costs of capital often, but not always, have positive net present values (NPV), and vice-versa. Investment projects with IRRs below their costs of capital often, but not always, have negative net present values (NPV), and vice-versa. If the project is a financing method rather than an ordinary investment project, these rules reverse.
- IRR suffers from comparison problems because it does not adjust for project scale. IRR can also be difficult to use if the cost of capital depends on the project cash flow timing.
- The profitability index is often acceptable, too. It rearranges the NPV formula. If used by itself, it often provides the same capital budgeting advice as

NPV. But, like IRR, the profitability index can make projects with lower upfront costs and scale appear relatively more desirable.

- The payback measure is commonly used. It suggests taking the projects that return the original investment most quickly. It discriminates against projects providing very large payments in the future. Although it sometimes provides useful information, it is best avoided as a primary decision rule.
- The information that many other capital-budgeting measures provide can sometimes be “interesting.” However, they often provide results that are not sensible and therefore should generally be avoided—or at least consumed with great caution.
- NPV and IRR are the methods most popular with CFOs. This makes sense. It remains a minor mystery as to why the payback method enjoys the popularity that it does.

### Keywords

CFO, 70. Chief financial officer, 70. Discounted payback, 68. Hurdle rate, 64. IRR, 59. Internal rate of return, 59. Payback rule, 68. Profitability index, 67. Scenario analysis, 57. Sensitivity analysis, 57. Separation of decisions, 57. YTM, 59. Yield-to-maturity, 59.

### Answers

**Q 4.1** The fact that you can use capital markets to shift money back and forth without costs allows you to consider investment and consumption choices independently.

**Q 4.2** If you invest \$400, the project will give  $\$400 \cdot 1.15 = \$460$  next period. The capital markets will value the project at  $\$460/1.10 \approx \$418.18$ . You should take the project and immediately sell it for \$418.18. Thereby, you will end up being able to consume  $\$500 - \$400 + \$418.18 = \$518.18$ .

**Q 4.3** The equation that defines IRR is Formula 4.1 on Page 59.

$$\text{Q 4.4} \quad -\$1,000 + \frac{\$1,000}{(1 + \text{IRR})} = 0 \implies \text{IRR} = 0\%$$

$$\text{Q 4.5} \quad -\$1,000 + \frac{\$500}{(1 + \text{IRR})} + \frac{\$500}{(1 + \text{IRR})^2} = 0 \implies \text{IRR} = 0\%$$

$$\text{Q 4.6} \quad -\$1,000 + \frac{\$600}{(1 + \text{IRR})} + \frac{\$600}{(1 + \text{IRR})^2} = 0 \implies \text{IRR} \approx 13.07\%$$

$$\text{Q 4.7} \quad -\$1,000 + \frac{\$900}{(1 + \text{IRR})} + \frac{\$900}{(1 + \text{IRR})^2} = 0 \implies \text{IRR} = 50\%$$

**Q 4.8** The spreadsheet function is called IRR(). The answer pops out as 15.5696%. Check:  $-\$100 + \$55/1.16 + \$70/1.16^2 \approx 0$ .

**Q 4.9** The coupon bond's YTM is 5%, because  $-\$1,000 + \frac{\$50}{1.05} + \frac{\$50}{1.05^2} + \frac{\$50}{1.05^3} + \frac{\$50}{1.05^4} + \frac{\$1,050}{1.05^5} = 0$ . The YTM of such a bond (annual coupons) is equal to the coupon rate when a bond is selling for its face value.

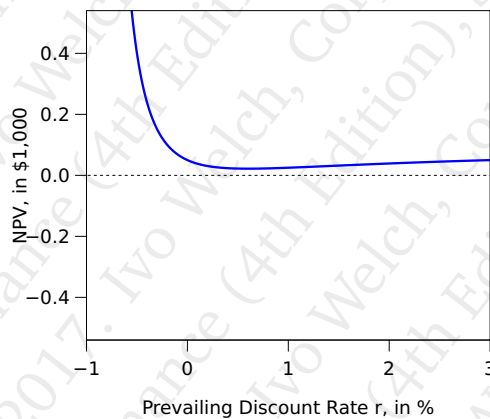
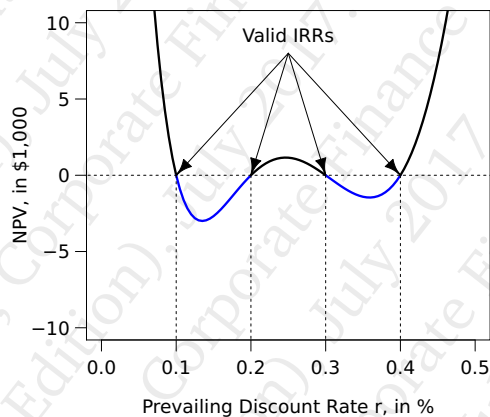
**Q 4.10** The YTM is 10%, because  $-\$1,000 + \$1,611/1.10^5 \approx 0$ .

**Q 4.11** You are seeking the solution to  $-\$25,000 + \frac{\$1,000}{(1 + \text{YTM})^1} + \frac{\$1,000}{(1 + \text{YTM})^2} + \frac{\$25,000}{(1 + \text{YTM})^2} = 0$ . It is  $\text{YTM} = 4\%$ .

**Q 4.12** For example,  $C_0 = -\$100, C_1 = +\$120, C_2 = -\$140, C_3 = +\$160, C_4 = -\$20$ . (The solutions are  $IRR \approx -85.96\%$  and  $IRR \approx +9.96\%$ . The important aspect is that your example has multiple inflows and multiple outflows.)

**Q 4.13** For example,  $C_0 = -\$100, C_1 = -\$200, C_2 = -\$50$ . No interest rate can make their present value equal to zero, because all cash flows are negative. This project should never be taken, regardless of cost of capital.

**Q 4.14** For projects A and B, the valid IRRs are 10%, 20%, 30%, and 40%. The plot for A follows. The plot for B has a y-scale that is 50 times larger. For project C, there is no IRR, also shown in the plot below.



For projects D, E, and F, the IRR is 100%. For project G, the IRRs are 0% and 100%.

**Q 4.15** The (unique) IRR is 56.16%. This is higher than your 30% cost of capital, so you should take this project. The NPV is  $+\$1,057.35$ . Because this is positive, it gives the same recommendation—accept.

**Q 4.16** The IRR is 19.73%. This is lower than your 20% cost of capital, so you should not take this project. The NPV is  $-\$23.92$ . IRR and NPV agree on the reject recommendation.

**Q 4.17** The IRR is 8.44%. This is above the prevailing interest rate. However, the cash flows are like that of a financing project.

This means that it is a negative NPV project of  $-\$7.71$ . You should not take it.

**Q 4.18** (1) The IRR-maximizing investment choice of  $C_0$  is an epsilon. The IRR is then close to infinity. The NPV is 0. (2) The NPV-maximizing (and best) choice is an investment of  $\$226,757$ . This also happens to be the project's NPV. The IRR is 110%.

**Q 4.19** The problems are (a) you need to get the sign right to determine whether you should accept the project above or below its hurdle rate; (b) you need to make sure you have only one unique IRR (or work with a more complicated version of IRR, which we have not done); (c) you cannot use it to compare different projects that have different scales; and (d) you must know your cost of capital.

**Q 4.20** Project A has a positive NPV of

$$\begin{aligned} & \$50,000 + \frac{-\$250,000}{1.25} + \frac{\$467,500}{1.25^2} \\ & + \frac{-\$387,500}{1.25^3} + \frac{\$120,120}{1.25^4} \approx \$1.15 \end{aligned}$$

Project B has an NPV of  $-\$1.15$ . You should take project A, but not B. If you plot the NPV as a function of the interest, you will see that there are multiple IRRs for these projects, specifically at 10%, 20%, 30%, and 40%. With a cost of capital of 25%, you cannot easily determine which of these two projects you should take. Make your life easy, and just use NPV instead.

**Q 4.21** Project A has an NPV of

$$\begin{aligned} & +\$500,000 + \frac{-\$200,000}{1.25} + \frac{-\$200,000}{(1.25)^2} + \frac{-\$200,000}{(1.25)^3} \\ & = \$109,600 \end{aligned}$$

It has an IRR of 9.70%. Project B has an NPV of  $\$70,000$ , and no IRR (it is always positive). Therefore, even though the second project should be taken for any interest rate—which is not the case for the first—the first project is better. Take project A.

**Q 4.22** The first project (A) has an NPV of  $\$20.66$  and an IRR of 13.07%. The second project (B) has an NPV of  $\$2.07$  and the same IRR of 13.07%. The third project (C) has an NPV of  $\$10.74$  and an IRR of 25.69%. Even though project A does not have the highest IRR, you should take it.

**Q 4.23** The IRR is 6.81%. This is between the one-year 5% and the two-year 10% interest rates. Therefore, the IRR capital-budgeting rule cannot be applied. The NPV rule gives you  $-\$1,000 + \$600/1.05 + \$500/1.155 \approx \$4.33$ , so this is a good project that you should take.

**Q 4.24** The first project (A) has present values of future cash flows of  $\$520.66$ ; the second (B) of  $\$52.07$ ; the third (C) of  $\$60.74$ . The profitability indexes are  $\$520.66/\$500 \approx 1.04$ ,  $\$52.07/\$50 \approx 1.04$ , and  $\$60.74/\$50 \approx 1.21$ . Nevertheless, you should go with the first project, because it has the highest net present value. The discrepancy between the NPV and the profitability rule recommendations is because the latter does not take project scale into account.

## End of Chapter Problems

**Q 4.25.** Given the same NPV, would you be willing to pay extra for a project that bears fruit during your lifetime rather than after you are gone?

**Q 4.26.** How bad a mistake is it to misestimate the cost of capital in a short-term project? Please illustrate.

**Q 4.27.** How bad a mistake is it to misestimate the cost of capital in a long-term project? Please illustrate.

**Q 4.28.** What is the difference between YTM and IRR?

**Q 4.29.** A project has cash flows of  $-\$1,000$ ,  $+\$600$ , and  $+\$300$  in consecutive years. What is the IRR?

**Q 4.30.** What is the YTM of a standard 6% level semiannual 10-year coupon bond that sells for its principal amount today (i.e., at par =  $\$100$ )?

**Q 4.31.** A coupon bond costs  $\$100$ , then pays  $\$10$  interest each year for 10 years, and pays back its  $\$100$  principal in 10 years. What is the bond's YTM?

**Q 4.32.** A project has cash flows  $-\$100$  as of now,  $+\$55$  next year, and  $+\$60.50$  in the year after. How can you characterize the "rate of return" (loosely speaking) embedded in its cash flows?

**Q 4.33.** Under what circumstances is an IRR a rate of return? Under what circumstances is a rate of return an IRR?

**Q 4.34.** Give an example of a problem that has multiple IRR solutions.

**Q 4.35.** Your project has cash flows of  $-\$1,000$  in year 0,  $+\$3,550$  in year 1,  $-\$4,185$  in year 2, and  $+\$1,638$  in year 3. What is its IRR?

**Q 4.36.** Your project has cash flows of  $-\$1,000$  in year 0,  $+\$3,550$  in year 1,  $-\$4,185$  in year 2, and  $-\$1,638$  in year 3. What is its IRR?

**Q 4.37.** A project has cash flows of  $+\$400$ ,  $-\$300$ , and  $-\$300$  in consecutive years. The prevailing interest rate is 5%. Should you take this project?

**Q 4.38.** A project has cash flows of  $-\$100$ ,  $+\$55$ , and  $+\$60.50$  in consecutive years starting from right now. If the hurdle rate is 10%, should you accept the project?

**Q 4.39.** If a project has a cash inflow of  $\$1,000$  followed by cash outflows of  $\$600$  in two consecutive years, then under what discount rate scenario should you accept this project?

**Q 4.40.** You can invest in a project with returns that depend on the amount of your investment. Specifically, the formula relating next year's payoff (cash flow) to your investment today is  $C_1 = \sqrt{-C_0 - \$0.1}$ , where  $C_0$  and  $C_1$  are measured in millions of dollars. For example, if you invest  $\$500,000$  in the project today, it will return  $\sqrt{\$0.5 - \$0.1} \approx \$0.632$  million next year. The prevailing interest rate is 6% per annum. Use a spreadsheet to answer the following two questions:

1. What is the IRR-maximizing investment choice of  $C_0$ ? What is the NPV at this level?
2. What is the NPV-maximizing investment choice of  $C_0$ ? What is the IRR at this level?

**Q 4.41.** The prevailing interest rate is 10%. If the following three projects are mutually exclusive, which should you take?

	Y0	Y1	Y2
A	+\$500	-\$300	-\$300
B	+\$50	-\$30	-\$30
C	+\$50	-\$35	-\$35

What does the NPV rule recommend? What does the IRR rule recommend?

**Q 4.42.** What are the profitability indexes and the NPVs of the following two projects: project A that requires an investment of  $\$5$  and gives  $\$20$  per year for three years, and project B that requires an investment of  $\$9$  and gives  $\$25$  per year for three years? The interest rate is 10%. If you can invest in only one of the projects, which would you choose?

**Q 4.43.** Consider the following project:

Year	Y0	Y1	Y2	Y3	Y4	Y5	Y6
Cash Flow	$-\$10$	$\$5$	$\$8$	$\$3$	$\$3$	$\$3$	$-\$6$

1. What is the IRR?
2. What is the payback time?
3. What is the profitability index?

Q 4.44. Consider the following project:

	Y0	Y1	Y2	Y3	Y4	Y5	Y6	Y7
CF	\$0	-\$100	\$50	\$80	\$30	\$30	\$30	-\$60

1. What is the IRR?
2. What is the payback time?
3. What is the profitability index?

Q 4.45. The prevailing cost of capital is 9% per annum. What would various capital-budgeting rules recommend for the following projects?

	Y0	Y1	Y2	Y3	Y4
A	-\$1,000	\$300	\$400	\$500	\$600
B	-\$1,000	\$150	\$200	\$1,000	\$1,200
C	-\$2,000	\$1,900	\$200		
D	-\$200	\$300			
E	-\$200	\$300	\$0	-\$100	

Q 4.46. What are the most prominent methods for capital budgeting in the real world? Which make sense?



### Data and Programming for Masters Students

**Task:** Write a program that reads a set of cash flows from a file and calculates the IRR(s). Extra credit for the ability to detect (some or all) multiple IRRs and/or non-existing IRRs (and that detects them efficiently).