6

Uncertainty, Default, and Risk

Risk-neutral Promised versus Expected Returns; and Debt versus Equity

You are now entering the world of uncertainty and abandoning the pleasant idea that you have perfect foresight. We shall still pretend, however, that you live in a perfect market with no taxes, no transaction costs, no differences of opinion, and infinitely many investors and firms. But you will learn in this chapter that the presence of uncertainty adds quite a bit of additional complexity and realism.

Net present value still rules supreme, but you will now have to face the sad fact that it is no longer easy to use. It is *not* the NPV concept that is difficult. Instead, it is the *inputs* that are difficult—the expected cash flows and appropriate costs of capital that you now have to guesstimate.

In a world of uncertainty, there are scenarios in which you will get more cash than you expected and scenarios in which you will get less. The single most important insight under uncertainty is that you must always draw a sharp distinction between *promised* (or *quoted* or *stated*) returns and *expected* returns. Because firms can default on payments or go bankrupt in the future, expected returns are lower than promised returns.

After some necessary statistical background, this chapter will cover two important finance topics: First, you must learn how much lenders should charge borrowers if there is the possibility of default. Second, you must learn how to work with the two building blocks of financing—namely, debt and equity.

6.1 An Introduction to Statistics

Statistics is about characterizing an uncertain world.

Statistics has the reputation of being the most painful of the foundation sciences for finance—but you absolutely need to understand it to describe an uncertain future. Yes, it can be a difficult subject, but if you have ever placed a bet in the past, chances are that you already have a good intuitive grasp of what you need. In fact, I had already sneaked the term "expected" into previous chapters, even though it is only now that this book covers what this precisely means.

Random Variables and Expected Values

The "expected value" is the average outcome if the random draw is repeated infinitely often. It need not be a possible realization.

The most important statistical concept is the **expected value**, which is the probability-weighted average of all possible outcomes. It is very similar to a **mean** or **average**. The difference is that the latter two names are used if you work with *past* outcomes, while the expected value applies if you work with *future* outcomes. For example, say you toss a coin, which can come up either heads or tails with equal probability. You receive $1 if the coin comes up heads and $2 if the coin comes up tails. Because you know that there is a 50% chance of $1 and a 50% chance of

Ivo Welch, Corporate Finance (4th Edition), July 2017.

$2, the expected value of each coin toss is $1.50. If you repeated this infinitely often, and if you recorded the series of **realizations** (actual outcomes), the mean would converge to exactly $1.50. Of course, in any one throw, $1.50 can never come up—the expected value does not need to be a possible realization of a single coin toss.

IMPORTANT

> The expected value is just the mean (a fancy word for average) if you could repeat an experiment (the random draws) infinitely often.

A random variable is a number whose realization is not yet known.

To make it easier to work with uncertainty, statisticians have invented the concept of the **random variable**. It is a variable whose outcome has not yet been determined. In the coin toss example, you can define a random variable named c (for "coin toss outcome") that takes the value $1 with 50% probability and the value $2 with 50% probability. (Random variables are often written with tildes over them, such as $\tilde{c}$, but we will dispense with this formality in our book.) The expected value of c is $1.50. To denote the expected value, we use the notation $\mathcal{E}$. In this bet,

$$\begin{array}{ccccccc} \mathcal{E}(c) & = & 50\% \cdot \$1 & + & 50\% \cdot \$2 & = \$1.50 \\ \text{Expected Value(Coin Toss)} & = & \text{Prob(Heads)} \cdot \$1 & + & \text{Prob(Tails)} \cdot \$2 & \end{array}$$

After the coin has been tossed, the actual outcome c could, for example, be c = $2. After the toss, this c is no longer a random variable. Also, if you are certain about the outcome, perhaps because there is only one possible outcome, then the actual realization and the expected value are the same. The random variable is then the same as an ordinary nonrandom variable. Is the expected outcome of the coin toss a random variable? No: You know the expected outcome is $1.50 even before the toss of the coin. The expected value is known; the uncertain outcome is not. The expected value is an ordinary nonrandom variable; the possible outcome is a random variable. Is the outcome of the coin throw *after* it has come down heads a random variable? No: It is an actual outcome and you know what it is (heads), so it is no longer a random variable.

A random variable is a statistical distribution.

A random variable is defined by the **probability distribution** of its possible outcomes. The coin throw distribution is simple: the value $1 with 50% probability and the value $2 with 50% probability. This is sometimes graphed in a **histogram**, which is a graph that has the possible outcomes on the x-axis and the frequency (or probability) on the y-axis. Exhibit 6.1 shows the histogram for the coin throw. In fact, you can think of a random variable as a placeholder for a histogram.

A final note—perfect markets.

One final reminder: In this chapter, we are eliminating our certainty assumption. But we are *not* (yet) eliminating our perfect market assumption. The assumption of no-disagreement means that we all must agree on the probabilities of all possible outcomes. An example of an imperfect market would be if you believed that there was a 51% probability of an outcome of $1, and I believed there was a 50% probability of $1.

Fair Bets

An example with three possible outcomes.

A **fair bet** is a bet that costs its expected value. If repeated infinitely often, both the person offering the bet and the person taking the bet would expect to end up even. For example, call D your payoff based on the following structure:

- There is a 1/4 chance that you will be paid $2;
- a 1/4 chance that you will be paid $10;
- and a 2/4 chance that you will be paid $8.

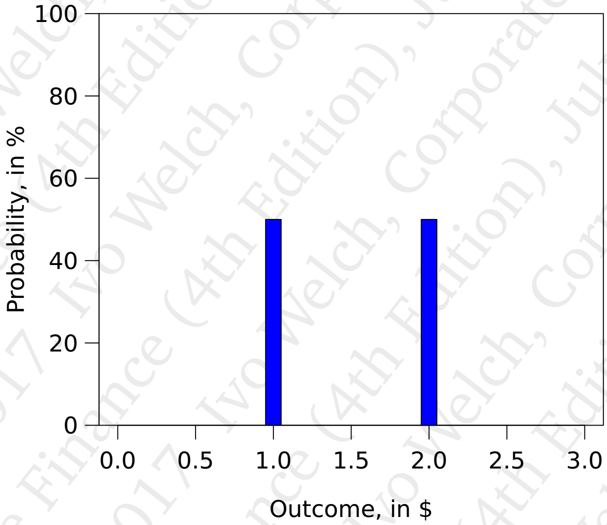

Exhibit 6.1: *A Histogram for a Random Variable with Two Equally Likely Outcomes, \$1 and \$2.*

You can simulate this payoff structure by drawing a card from a complete deck. If it is ♣, you get a value V of \$2; if it is ♦, you get \$10, and if it is ♥or ♠, you get \$8. What would be a fair price for this card bet? The uncertain payoff is a random variable. Let's call it D. First, you must determine $\mathrm{E}(\mathrm{D})$. It is

$$\begin{aligned}
\mathrm{E}(\mathrm{D}) &= \tfrac{1}{4} \cdot \$2 + \tfrac{1}{4} \cdot \$10 + \tfrac{2}{4} \cdot \$8 = \$7 \\
\mathrm{E}(\mathrm{D}) &= \mathrm{Prob}(\clubsuit) \cdot \mathrm{V}_{\clubsuit} + \mathrm{Prob}(\diamondsuit) \cdot \mathrm{V}_{\diamondsuit} + \mathrm{Prob}(\heartsuit \text{ or } \spadesuit) \cdot \mathrm{V}_{\heartsuit \text{ or } \spadesuit}
\end{aligned}$$

If you repeat this bet a zillion times, you would expect to earn \$7 zillion. On average, each bet would earn \$7, although some sampling variation in actual trials would make this a little more or a little less. If it costs \$7 to buy each single bet, it would be fair.

Generally, the procedure to compute expected values is always the same: Multiply each outcome by its probability and add up all these products.

The expected value is the probability-weighted sum of all possible outcomes.

$$\begin{aligned}
\mathrm{E}(\mathrm{X}) = &\quad \mathrm{Prob}(\text{First Possible Outcome}) \cdot \text{Value of First Possible Outcome} \\
&+ \mathrm{Prob}(\text{Second Possible Outcome}) \cdot \text{Value of Second Possible Outcome} \\
&+ \quad \vdots \\
&+ \mathrm{Prob}(\text{Last Possible Outcome}) \cdot \text{Value of Last Possible Outcome}
\end{aligned}$$

This is the formula that you used above,

$$\begin{aligned}
\mathrm{E}(\mathrm{D}) &= \tfrac{1}{4} \cdot \$2 + \tfrac{1}{4} \cdot \$10 + \tfrac{2}{4} \cdot \$8 = \$7 \\
&= \text{Sum of } [\mathrm{Prob}(\text{Each Outcome}) \times \text{Value of Each Outcome}]
\end{aligned}$$

Note that the formula is general. It works even with outcomes that are impossible. You would just assign probabilities of zero to them.

IMPORTANT

You must understand the following:

1. The difference between an ordinary variable and a random variable
2. The difference between a realization and an expectation
3. How to compute an expected value, given probabilities and outcomes
4. What a fair bet is

Q 6.1. Is the expected outcome (value) of a die throw a random variable?

Q 6.2. Could it be that the expected value of a bet is a random variable?

Q 6.3. For an ordinary die, assume that the random variable is the number on the die times two. Say the die throw came up with a "six" yesterday. What was its expected outcome before the throw? What was its realization?

Q 6.4. A stock that has the following probability distribution (outcome P_{+1}) costs \$50. Is an investment in this stock a fair bet?

Prob	P_{+1}	Prob	P_{+1}	Prob	P_{+1}	Prob	P_{+1}
5%	\$41	20%	\$45	20%	\$58	5%	\$75
10%	\$42	30%	\$48	10%	\$70		

Variance and Standard Deviation

We will measure the "reward" as the expected value. Looking ahead, the standard deviation is the most common measure of "risk" (spread).

In finance, we often need to measure the (average) **reward** that you expect to receive from making an investment. Ordinarily, we use the expected value of the investment as our measure of reward. We also often need to measure a second characteristic of an investment, its **risk**. Thus, we also need summary measures of how spread out the possible outcomes are. These two concepts will play starring roles in the next few chapters, where you will explore them in great detail. For now, if you are curious, think of risk as a measure of the variability of outcomes around your expected mean. The most common measure of risk is the standard deviation, which takes the square root of the sum of squared deviations from the mean—a mouthful. Let's just do it once for our card-draw problem. Recall our formula: the expectation are probability-weighted values. First, work out each squared deviation from the mean:

(Computing the variance can be a demeaning task.)

The first outcome is \$2. The mean is \$7, so the deviation from the mean is $\$2 - \$7 = -\$5$. You need the squared deviation from the mean, which is $(-\$5)^2 = +\$\$25$. The units are strange—dollars squared—and impossible to interpret intuitively. Don't even try.

The second outcome is \$10, so the deviation from the mean is $\$10 - \$7 = +\$3$. You need the squared deviation from the mean, which is $(+\$3)^2 = +\$\$9$.

The third outcome is \$8, so the deviation from the mean is $\$8 - \$7 = +\$1$. You need the squared deviation from the mean, which is $(\$1)^2 = +\$\$1$.

Together, in one table, this is

Probability	1/4	1/4	2/4
Outcome	\$2	\$10	\$8
Net of the Mean (\$7)	– \$5	+ \$3	\$1
Squared Net of Mean	\$\$25	\$\$9	\$\$1

Now compute the expected value of these squared deviations, which is called the **variance**:

$$\text{Var}\left(\text{Card Pay}\right) \;=\; 1/4 \cdot (\$\$25) + 1/4 \cdot (\$\$9) + 2/4 \cdot \$\$1 \;=\; \$\$9$$

The **standard deviation** is therefore

$$\text{Sdv}\left(\text{Card Pay}\right) \;=\; \sqrt{\$\$9} \;\approx\; \$3$$

There you have it—our mouthful: The standard deviation is the square root of the average squared deviation from the mean. Unlike the variance, the standard deviation has sensible units. Together, the mean and standard deviation allow you to characterize your bet. It is common phrasing, though a bit loose, to state that you expect to earn \$7 (the expected value) from a single card draw, plus or minus \$3 (the standard deviation).

Q 6.5. Reconsider the stock investment from Question 6.4. What is its risk—that is, what is the standard deviation of its outcome P_{+1}?

Risk Neutrality (and Preview of Risk Aversion)

Choosing investments only on the basis of expected values is assuming risk neutrality.

Fortunately, the expected value is all you need to learn about statistics for this chapter. This is because we are assuming—only for learning purposes—that everyone is **risk-neutral**. Essentially, this means that investors are willing to write or take any fair bet. For example, if you are risk-neutral, you would be indifferent between getting \$1 for sure and getting either \$0 or \$2, each with 50% probability. And you would be indifferent between earning 10% from a risk-free bond and earning either 0% or 20%, again with fifty-fifty probability, from a risky bond. You have no preference between investments with equal expected values, no matter how safe or uncertain these investments may be.

Risk aversion means you would prefer the safe project. Put differently, you would demand an extra "kicker" to take the riskier project instead.

If, instead, you are **risk-averse**, you would not want to invest in the more risky alternative if both the risky and safe alternatives offered the same expected rate of return. You would prefer the safe \$1 to the unsafe \$0 or \$2 investment. You would prefer a 10% risk-free bond to the unsafe corporate bond that would pay either 0% or 20%. In this case, if I wanted to sell you a risky project or a risky bond, I would have to offer you a higher expected rate of return as risk compensation. I might have to pay you, say, 5 cents to get you to be willing to accept the project that pays off \$0 or \$2 if you can instead earn \$1 elsewhere. Alternatively, I would have to lower the price of my corporate bond so that it offers you a higher expected rate of return, say, 1% or 21% instead of 0% or 20%.

For a given investor, bigger bets usually require more compensation for risk.

It is true that if you are risk-averse, you should not accept fair bets. (You can think of this as the definition of risk aversion.) But would you really worry about a bet for either +\$1 or –\$1? Probably not. For small bets, you are probably close to risk-neutral—I may not have to pay you even 1 cent extra to induce you to take this bet. But what about a bet for plus or minus \$100? Or for plus or minus \$10,000? My guess is that you would be fairly reluctant to accept the latter bet without getting extra compensation for risk bearing. If you are like most investors, you are more risk-averse when the bet is larger. To take the plus or minus \$10,000 bet, I would probably have to offer you several hundred dollars extra.

Corporate Finance (4th Edition), July 2017. Ivo Welch

Financial markets can spread risk and thereby lower the aggregate risk aversion.

However, your own personal risk aversion is not what matters in financial markets. Instead, the financial markets set investments prices in line with the market's aggregate risk aversion. The reason is risk sharing. For example, if you could share the $10,000 bet with 10,000 other students in your class, your own part of the bet would be only plus or minus $1. And some of your colleagues may be willing to accept even more risk for relatively less extra risk compensation—they may have healthier bank accounts or wealthier parents. Therefore, when you can lay bets across many investors, the effective risk aversion of the group will be lower than that of any of its members. And this is exactly how financial markets work: Their aggregate risk absorption capabilities are considerably higher than those of their individual investors. In effect, the financial markets are less risk-averse than individual investors.

The tools you learn now will remain applicable under risk aversion.

You will study risk aversion in the next chapters. In this chapter, we will focus on pricing under risk neutrality. But, as always, all tools you learn in this simpler scenario will remain applicable in the more complex scenario in which investors are risk-averse. Moreover, in the real world, the differences between promised and expected returns that are discussed in this chapter are often more important (in terms of value) than the extra compensation for risk aversion that is ignored in this chapter.

Q 6.6. Are investors more risk-averse for small bets or for large bets? Should "small" be defined relative to investor wealth?

Q 6.7. Can the aggregate financial market be less risk-averse than each of its individual investors?

6.2 Interest Rates and Credit Risk (Default Risk)

Risk-free and risky lending.

Most loans in the real world are not risk-free, because the borrower may not fully pay back what was promised. We will assume that there is one exception, which is that U.S. Treasuries are risk-free loans in nominal terms. In principle, the United States can always tax more and print more dollars to satisfy all promised bond payments. Therefore, it is reasonable to assume the United States cannot default. (Intelligent people can disagree. Washington politics is so dysfunctional that the U.S. may actually default not for lack of dollars, but by choice.) So, how do you compute appropriate expected rates of return for risky bonds?

The Ruin of the First Financial System

The earliest known example of widespread financial default occurred in the year 1788 B.C.E., when King Rim-Sin of Uruk (Mesopotamia) repealed *all* loan repayments. The royal edict effectively destroyed a system of flourishing commerce and finance, which was already many thousands of years old! It is not known why Rim-Sin did so. Interest rates were modest, roughly 4% per annum for five-year loans.

William Goetzmann, Yale University

Risk-Neutral Investors Demand Higher Promised Rates

If my repayment is certain, you should charge me the same interest rate that the U.S. Treasury offers.

Now, put yourself into the position of a banker. Assume that a 1-year Treasury note offers a safe annual rate of return of 10%. Your immediate problem is that you are contemplating making a 1-year loan of $1 million to me. What interest rate should you charge me on the loan? If you are 100% certain that I will fully pay the agreed-upon amount, you can just charge me 10%. You earn just as much from me as from the Treasury note. Both will pay back $1,100,000.

However, in the real world, there are few borrowers for whom you can be 100% certain that they will fully repay a loan. For example, assume you believe there is only a 50% chance that I will pay back the principal plus interest. (If I do pay it back, I will be called **solvent**). There is also a 50% chance that I will **default** (fail to pay all that I have promised). This is often informally called bankruptcy. In this case, I may only be able to pay back \$750,000—all that I have left. If, as the bank, you were to charge me a 10% interest rate, your expected payout would be

If you quote me the same interest rate, you would expect to earn a lower interest rate if there is a chance of default.

$$\begin{array}{ccccccccc} 50\% & \cdot & \$750{,}000 & + & 50\% & \cdot & \$1{,}100{,}000 & = & \$925{,}000 \\ \text{Prob}(\text{Default}) & \cdot & (\text{Pay if Default}) & + & \text{Prob}(\text{Solvent}) & \cdot & (\text{Pay if Solvent}) & & \end{array}$$

Your *expected* return would not be \$1,100,000, but only \$925,000. Your *expected* rate of return would not be +10%, but only $\$925,000/\$1,000,000 - 1 = -7.5\%$. Extending such a loan would not be—pardon the pun—in your best interest: You can do better by investing your \$1,000,000 into government Treasury notes.

A Short History of Bankruptcy

The framers of the United States Constitution had the English bankruptcy system in mind when they included the power to enact "uniform laws on the subject of bankruptcies" in Article I (powers of the legislative branch). The first bankruptcy law, passed in 1800, virtually copied the existing English law. Our bankruptcy laws thus have their conceptual origins in English bankruptcy law prior to 1800. On both sides of the Atlantic, however, much has changed since then.

Early English law had a distinctly pro-creditor orientation and was noteworthy for its harsh treatment of defaulting debtors. Imprisonment for debt was the order of the day, from the time of the Statute of Merchants in 1285 until Charles Dickens's time in the mid-nineteenth century. (In fact, when Dickens was a child, his father spent time in debtor's prison.) The common law *Writs of Capias* authorized "body execution," that is, seizure of the body of the debtor, to be held until payment of the debt.

English law was not unique in its lack of solicitude for debtors. History's annals are replete with tales of harsh treatment of debtors. Punishments inflicted upon debtors included forfeiture of all property, relinquishment of the consortium of a spouse (think about this one!), imprisonment, and death. In Rome, creditors were apparently authorized to carve up the body of the debtor. However, scholars debate the extent to which the letter of that law was actually enforced.

Charles Jordan Tabb, 1995, "The History of the Bankruptcy Laws in the United States."

You should conclude that you must demand a higher interest rate from risky borrowers as a banker, even if you just want to "break even" (i.e., expect to earn the same \$1,100,000 that you could earn in Treasury notes). If you solve

You must ask for a higher promised interest—received only in good times—in order to make up for my default risk.

$$\begin{array}{ccccccccc} 50\% & \cdot & \$750{,}000 & + & 50\% & \cdot & (\text{Promised Repayment}) & = & \$1{,}100{,}000 \\ \text{Prob} & \cdot & (\text{Payment if Default}) & + & \text{Prob} & \cdot & (\text{Payment if Solvent}) & = & \text{Treasury Payment} \end{array}$$

for the desired promised repayment, you will find that you must ask me for \$1,450,000. The promised interest rate is therefore $\$1{,}450{,}000/\$1{,}000{,}000 - 1 = 45\%$. Of this 45%, 10% is the **time premium** that the Treasury pays. Therefore, you can call the remaining 35% the **default premium**—the difference between the promised rate and the expected rate that you, the lender, would have to demand just to break even. It is very important that you realize that the default premium is not extra compensation for your taking on more risk, say, relative to holding Treasuries. You don't receive any such extra compensation in a risk-neutral world. The default premium just fills the gap between the expected return and the promised return.

You are always quoted promised returns, and not expected returns. The risk is called "credit risk."

➤ IRR, YTM, Sect. 4.2, Pg.59.

You rarely observe expected rates of return directly. Newspaper and financial documents almost always provide only the **promised interest rate**, which is therefore also called the **quoted interest rate** or the **stated interest rate**. When you read a published yield-to-maturity, it is also usually only a promised rate, not an expected rate—that is, the published yield is an internal rate of return that is calculated from promised payments, not from expected payments. Of course, you should never make capital budgeting decisions based on promised IRRs. You almost always want to use an expected IRR (YTM). But you usually have easy access only to the promised rate, not the expected rate. On Wall Street, the default premium is often called the **credit premium**, and **default risk** is often called **credit risk**.

Q 6.8. For what kind of bonds are expected and promised interest rates the same?

A More Elaborate Example With Probability Ranges

Again, I sometimes may not be able to repay.

This distinction between expected and promised rates is so important that it is worthwhile to work another more involved example. Assume again that I ask you to lend me money. You believe that I will pay you what I promise with 98% probability; that I will repay half of what I borrowed with 1% probability; and that I will repay nothing with 1% probability. I want to borrow $200 from you, which you could alternatively invest into a government bond promising $210 (i.e., a 5% interest rate). What interest rate would you ask of me?

If you ask me to pay the risk-free interest rate, you will on average earn less than the risk-free interest rate.

If you ask me for a 5% interest rate, next year (time 1), your $200 investment today (time 0) will produce the following:

Payoff (C_1)	Rate of Return (r)	Frequency (Prob)
$210	+5.0%	98% of the time
$100	–50.0%	1% of the time
$0	–100.0%	1% of the time

Therefore, your expected payoff is

$$E(C_1) = 98\% \cdot \$210 + 1\% \cdot \$100 + 1\% \cdot \$0 = \$206.80$$
$$= \text{Prob} \cdot \text{Cash Flow} + \text{Prob} \cdot \text{Cash Flow} + \text{Prob} \cdot \text{Cash Flow}$$

Your expected return of $206.80 is less than the $210 that the government promises. Put differently, if I *promise* you a rate of return of 5%,

$$\text{Promised}(r) = \frac{\$210 - \$200}{\$200} = 5.00\%$$
$$\text{Promised}(r) = \frac{\text{Promised}(C_1) - C_0}{C_0}$$

then your expected rate of return would be only

$$E(r) = \frac{\$206.80 - \$200}{\$200} = 3.40\%$$
$$E(r) = \frac{E(C_1) - C_0}{C_0}$$

This is less than the 5% interest rate that Uncle Sam promises—and surely delivers.

You need to determine how much I have to promise you just to break even. You want to expect to end up with the same \$210 that you could receive from Uncle Sam. The expected loan payoff is the probability-weighted average payoff. You want this payoff to be not \$206.80 but the \$210 that you can earn if you invest your \$200 into government bonds. You need to solve for an amount x that you receive if I have money,

Let's determine how much more interest promise you need to break even.

$$\mathsf{E}(C_1) = 98\% \cdot x + 1\% \cdot \$100 + 1\% \cdot \$0 = \$210.00$$

The solution is that if I promise you $x \approx \$213.27$, you will expect to earn the same 5% interest rate that you can earn in Treasury notes. This \$213.27 for a cash investment of \$200 is a *promised* interest rate of

$$\mathsf{Promised}(r) \approx \frac{\$213.27 - \$200}{\$200} \approx 6.63\%$$

$$\mathsf{Promised}(r) = \frac{\mathsf{Promised}(C_1) - C_0}{C_0}$$

Such a promise provides the following:

Payoff (C_1)	Rate of Return (r)	Frequency (Prob)
\$213.27	+6.63%	98% of the time
\$100.00	–50.00%	1% of the time
\$0.00	–100.00%	1% of the time

This comes to an *expected* interest rate of

$$\mathsf{E}(r) \approx 98\% \cdot (+6.63\%) + 1\% \cdot (-50\%) + 1\% \cdot (-100\%) \approx 5\%$$

Q 6.9. Recompute the example from the text, but assume now that the probability of receiving full payment in one year on a \$200 investment of \$210 is only 95%, the probability of receiving \$100 is 1%, and the probability of receiving absolutely no payment is 4%.

1. At the promised interest rate of 5%, what is the expected interest rate?
2. What interest rate is required as a promise to ensure an expected interest rate of 5%?

Deconstructing Quoted Rates of Return—Time and Default Premiums

The difference of 1.63% between the promised (or quoted) interest rate of 6.63% and the expected interest rate of 5% is the default premium—it is the extra interest rate that is caused by the default risk. Of course, you only receive this 6.63% *if* everything goes perfectly. In our perfect market with risk-neutral investors,

The difference between the promised and expected interest rate in a risk-neutral perfect world is the default premium.

$$\underset{\text{“Promised Interest Rate”}}{6.63\%} = \underset{\text{“Time Premium”}}{5\%} + \underset{\text{“Default Premium”}}{1.63\%}$$

IMPORTANT

Except for 100%-safe bonds, the promised (or quoted) rate of return is higher than the expected rate of return. Never confuse the promised rate with the (lower) expected rate. If you only remember one thing from this book, this should be it!

Financial securities and information providers rarely, if ever, provide information about expected rates of return. They almost always provide only quoted rates of return.

In a perfect risk-neutral world, all securities have the same expected rate of return.

On average, the expected rate of return is the expected time premium plus the expected default premium. Because the *expected* default premium is zero *on average*,

$$\begin{aligned} \mathsf{E}(\text{Rate of Return}) &= \mathsf{E}(\text{Time Premium}) + \qquad 0 \\ &= \mathsf{E}(\text{Time Premium}) + \mathsf{E}(\text{Realized Default Premium}) \end{aligned}$$

If you want to work this out, you can compute the expected realized default premium as follows: You will receive $6.63\% - 5\% = 1.63\%$ in 98% of all cases; $-50\% - 5\% = -55\%$ in 1% of all cases (note that you lose the time premium); and $-100\% - 5\% = -105\%$ in the remaining 1% of all cases (i.e., you lose not only all your money, but also the time premium). Therefore,

$$\mathsf{E}(\text{Realized Default Premium}) \approx 98\% \cdot (+1.63\%) + 1\% \cdot (-55\%) + 1\% \cdot (-105\%) \approx 0\%$$

Warning: Additional premiums will follow later.

In addition to the 5% time premium and the 1.63% default premium, in the real world, there are also other premiums that we have not yet covered:

Risk premiums that compensate you with (even) *higher* expected rates of return for your willingness to take on risk. They will be the subject of Chapter 10.

Imperfect market premiums (e.g., liquidity premiums) that compensate you for future difficulties in finding buyers for your bonds. They will be the subject of Chapter 11.

In normal times, these premiums are typically much lower than time premiums and default premiums in a bond context.

Q 6.10. Is the expected default premium positive?

Credit Ratings and Default Rates

Bond rating agencies: The most important corporate credit ratings are from Moody's and Standard & Poor's.

To make it easier for lenders to judge the probability of default, a number of data vendors for credit ratings have appeared. For individuals, Experian, Transunion, and Equifax provide credit ratings—you should request a free credit report for yourself from the Federal Trade Commission if you have never seen one. For small companies, Dun & Bradstreet provides similar credit scores. For corporations, the two biggest credit rating agencies are **Moody's** and **Standard&Poor's** (**S&P**). (There are also other less influential ones, like *Duff and Phelps* and *Fitch*.) For a fee, these agencies rate the probability that the issuer's bonds will default. This fee depends on a number of factors, such as the identity of the issuer, the desired detail in the agencies' investigations and descriptions, and the features of the bond (e.g., a bond that will pay off within one year is usually less likely to default before maturity than a bond that will pay off in thirty years; thus, the former is easier to grade).

Investment Grade

	Best									Barely
Moody's	Aaa	Aa1	Aa2	Aa3	A1	A2	A3	Baa1	Baa2	Baa3
Standard & Poor's	AAA	AA+	AA	AA–	A+	A	A–	BBB+	BBB	BBB–

Non-Investment Grade (Speculative or "Junk")

	Speculative							in Default
Moody's	Ba1	Ba2	Ba3	B1	B2	B3	Caa1, Caa2, Caa3, Ca , C	D
Standard & Poor's	BB+	BB	BB–	B+	B	B–	CCC	D

Exhibit 6.2: *Bond Rating Categories Used by Moody's and Standard & Poor's.*

The most important grade distinction is "junk" versus "investment grade."

The credit rating agencies ultimately do not provide a whole set of default probabilities (e.g., 1% chance of 100% loss, 1.2% chance of 99% loss, etc.), but just an overall rating grade. Exhibit 6.2 shows the categories for Moody's and Standard & Poor's. It is then up to the lender to translate the rating into an appropriate compensation for default risk. The top rating grades are called **investment grade**, while the bottom grades are called **speculative grade** (or **junk grade**).

Conflicted Ratings (and the Great Recession).

Ratings have limited usefulness:

1. They do not consider common risk, wherein many bonds would default at the same time. This will be an important concept in the next few chapters. See, most bond buyers should care more about the (small) risk of all their bonds blowing up at the same time and care less about one small individual bond in their many-bond portfolios defaulting. But common risk assessments are *not* what rating agencies provide.
2. Unlike most other financial market experts, rating agencies are not liable for their ratings or perspectives even if they deliberately deceive investors. (The 2010 Dodd-Frank Act repealed this exemption, but the SEC has granted indefinite "no-action" relief for most ratings.)
3. The strangest aspect, however, is how the rating agencies are paid. They collect fees for rating securities by the investment banks—how critical would you be of their bond products in this case? Not surprisingly, although they need to maintain some independence and reputation, the agencies have also often been good game when it comes to being manipulated—some would even call it bribed. A good part of the **Great Recession** (the financial crisis of 2009), falls squarely on the shoulders of the rating agencies, which earned billions providing optimistic ratings for issues explicitly engineered by investment banks to have high ratings. And although they have taken some steps to improve the situation, the basic conflicts of interest are still there. When public attention will have moved on to another "issue of the day," chances are that the ratings will return back to business as usual.
4. Ratings change over time: after their issue, bonds move up or down with probabilities of about 3-10% each per year.

Corporate Finance (4th Edition), Ivo Welch, July 2017.

Nevertheless, despite all their flaws, they are a useful source of information for potential bond buyers.

Empirical Evidence on Default

Here are historical probabilities of bond defaults by credit ratings.

Ed Altman and his coauthors (from New York University) collected corporate bond statistics from 1971 to 2015. Exhibit 6.3 gives you a sketch of how likely default was. (It is standard to define default as missing at least one coupon payment. It is *not* complete ultimate non-payment.) The average default rate was about 3.5% per year—but the left plot shows that it was much higher in recessions, where defaults typically shot up to over 10%. For example, in the Great Recession, about 11% of bonds failed to pay. (In 2008, about half of all defaults were Lehman Brothers' bankruptcy.) By 2010, the worst seemed to have passed. In retrospect, the Great Recession financial crisis ended up "not so bad" for most public corporations. (Lucky!) The right plot shows that corporate bonds originally rated A or better rarely defaulted, even 10 years after issue. However, about half of all CCC junk bonds would fail to pay at least one coupon within the first five years of issue.

➤ Senior and Junior Bonds, Chapter 16, Pg.433.

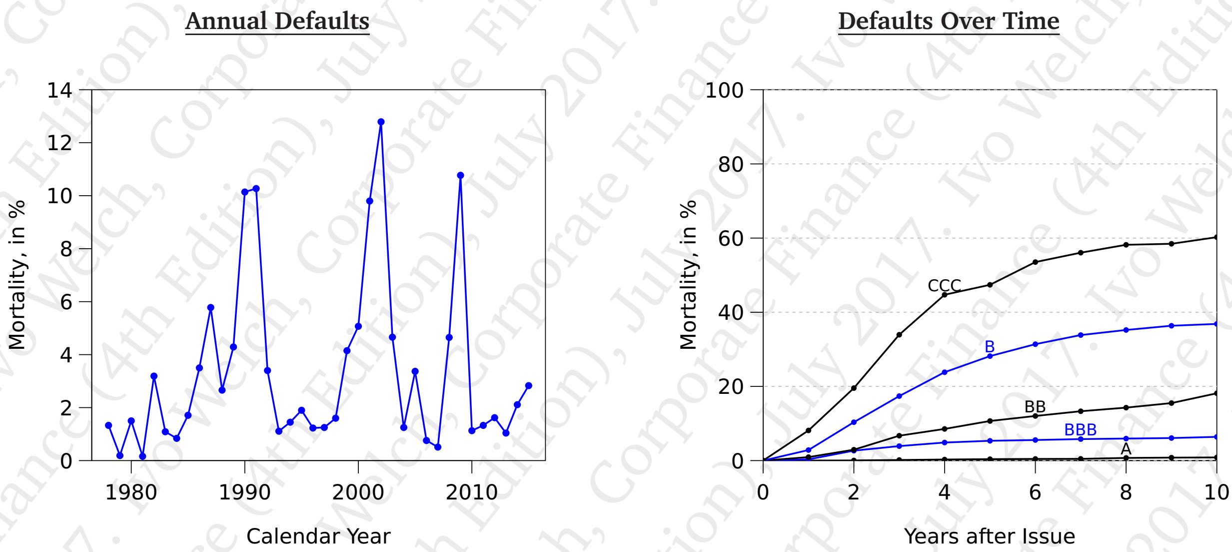

Exhibit 6.3: *Cumulative Historical Frequency of Default by Original Bond Rating, 1971–2015.* The left plot shows the rate at which bonds defaulted. For example, in 2009, about 11% of all corporate bonds failed to make at least one payment. The right plot shows the frequency of default within x years after issue, given the bond rating *at-issue* (not updated). For example, at some point during the first 7 years of their issue, about 1-in-3 bonds originally issued as B (poor) had not delivered on at least one promised bond payment. Corporate bonds originally rated A and better essentially did not default over their first 10 years. Source: Edward Altman and Brenda Kuehne, New York University, June 2016.

Moody's monthly *Default Report* lists recovery rates after bonds default. In 2005, they reported that from 1982 on, recovery rates were about 60% for senior secured bonds, 45% for senior unsecured bonds, and 30% for junior bonds. The typical recovery in a default was about 30-40 cents on the dollar, with 25 cents in recessions and 50 cents in booms. Low-rated bonds would pay less. These numbers seem to have remained similar in the 2010-2016 time period, too, but there is a lot of idiosyncratic variation across individual bonds.

Bond Contract Option Features

Before I show you how bonds are priced, I need to let you know that bonds in the real world differ from one another not just in credit risk. Most bonds have additional contract features that may also influence their quoted rates of return. For example, many corporate bonds allow the issuer to repay the loan early. (The same applies to almost all domestic mortgages.) If the interest rates in the future fall, this can be a good thing for the borrower and a bad thing for the lender. The borrower would pay off the loan and borrow more cheaply elsewhere. If the interest rates in the future rise, the borrower gets to pay just the earlier low interest rate. For example, assume that the interest rate is 10% today and you are lending me $90,909 in exchange for my promise to pay you $100,000 next year. One second after you extend the loan, one of two scenarios can happen:

Before I show you real-world quoted returns, I must explain that they can contain contract premiums.

1. The interest may fall to 5%. I would then simply repay your $90,909 loan and refinance at this lower interest rate elsewhere.
2. The interest rate may rise to 15%. In this case, I keep my $100,000 promise to pay next year—I received $90,909 for a loan that should have given me only $100,000/1.15 ≈ $86,957.

This would not be a good arrangement for you—unless you are appropriately compensated for giving me this option to prepay. Borrowers who want the right to repay without penalty therefore have to pay higher interest rates when they issue such bonds. Virtually all mortgage bonds in the United States allow prepayment and therefore carry higher interest rates than they would if they did not have a prepayment feature. Loosely speaking, you can classify these contract option features as default premiums, too, because on average they tend not to add or subtract from your expected rate of return. Sometimes they increase the amount paid, and sometimes they decrease the amount paid by the lender—just as a solvent bond would pay more to the lender and an insolvent bond would pay less to the lender.

Q 6.11. Does the historical evidence show that lower-grade borrowers default more often or that they pay less upon default?

Differences in Quoted Bond Returns in June 2016

So how do real-world credit risk and reflecting bond credit ratings translate into differences in promised (quoted) bond yields? Exhibit 6.4 lists the borrowing rates of various issuers on June 10, 2016 from Yahoo.

Historical rates of return: Riskier bonds indeed have higher stated rates of return.

The data looks broadly consistent with the theory—bonds that have higher default risk have to offer higher promised rates of return. Bonds with higher (better) credit ratings can find lenders at lower interest rates (higher bond prices).

Do lenders who extend loans to riskier creditors end up with higher average rates of return? This would be the case in a perfect market in which lenders and borrowers are risk-neutral. The evidence suggests that this is not exactly true, but it is also not as far from reality as naive readers would think. The majority of the investment-grade bond spread above the Treasury that you see in Exhibit 6.4 simply made borrowers and lenders come out about even. That is, the *expected* rates of return were much more similar to one another than the *promised* rates of return in the

Riskier bonds have to promise higher rates of return, but...

Ivo Welch, Corporate Finance (4th Edition), July 2017.

Type	2	5	10	20
	Years			
AAA	n/a	1.37	2.26	2.57
only Johnson&Johnson and Microsoft				
AA	0.78	1.52	2.43	3.58
e.g., Walmart, Apple, Intel				
U.S.	0.76	1.20	1.66	2.04

Type	2	5	10	20
	Years			
A	0.91	1.75	2.69	3.80
e.g., IBM, Morgan-Stanley, Target				
≈B		5% to 12%		
e.g., Tesla, Victoria's Secret.				
U.S.	0.76	1.20	1.66	2.04

Exhibit 6.4: *Promised Interest Rates and Treasury Bonds on June 9, 2016.* Source: Federal Reserve Economic Database, FRED, itself partially sourced from Moody's and Bank of America Merrill Lynch.

➤ De Jong-Driessen, Exhibit 11.1, Pg.272.

table suggest. If you put a gun to my head and asked me for an opinion, I would guess that about 80% of the promised spreads over Treasury are credit risk; about 10% are due to measurement or contract features (e.g., the timing of coupons or some option contingencies); and only about 10% are extra compensation that the creditors earn on average above and beyond what they would earn in equivalent U.S. Treasuries. (And, of course, all interest income is taxable.)

Historical Yields of Corporate Bonds

You already saw in Exhibit 6.3 that actual defaults by reasonably large corporations were cyclical and increased only briefly in the Great Recession. Similarly but even more extreme, the interest rates that lower-rated corporations paid (which are *promise*d rates that reflect expectations of non-payment over the full life of the bond) were cyclical and spiked in 2009. Exhibit 6.5 plots the historical yields of the 20-year Treasury bond, of Moody's-rated Aaa and Baa investment-grade bond portfolios, and non-investment grade (high-yield) bond portfolios. The typical investment-grade bond promised about 100-200 basis points above the Treasury, while the typical junk bond promised about 200-600 basis points above the Treasury. (Junk is relative—non-publicly traded corporations usually pay even higher rates on non-collateralized obligations.) Any non-investment grade corporation that had to borrow during the Great Recession was in trouble. Promised interest rates were more than 2,000 bp higher than Treasuries, as investors were fleeing to safety. While investment-grade bonds have pretty much returned to normal by 2016, non-investment grade issuers continue to have to promise relatively high spreads.

Dilbert on libor: 2012-10-04

Other Rates

We could discuss risk and reward for many other types of credit. Credit risks are not always similar. Mortgage investors in Arizona can face different interest rates than London banks. The latter is called the London Interbank Offered Rate (**Libor**). Incidentally, Libor plays an important role, because it is also a commonly used benchmark for about $350 trillion of derivatives. Even your mortgage interest rate may be tied to Libor.

Ivo Welch, Corporate Finance (4th Edition), July 2017.

The Libor Scandal of 2008

The dependence of many other financial contracts on the daily quoted Libor rate made its active manipulation by large banks extraordinarily profitable. This illegal manipulation seemed to have begun in about 1991 and lasted until a WSJ investigation discovered and wrote about it in 2008. As usual, the traders kept their bonuses, while the fairly innocent shareholders of the major banks had to pay many billions of dollars in penalties.

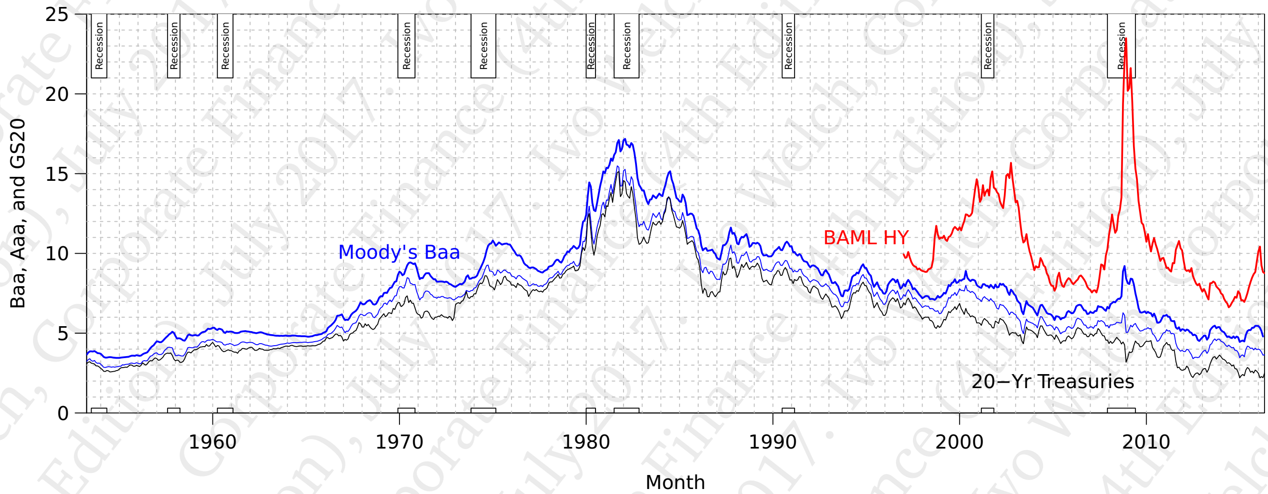

Exhibit 6.5: *Yields on 20-Year Treasuries (black) and Promised Yields on Corporate Bonds, 1955–2016.* The plot shows that the typical Baa corporate bond, in blue, spread over the 20-year Treasury was about 150 bp (ranging from about 40 to about 500 bp). Aaa bond yields were between those of the long-term Treasury and the Baas. In contrast, high-yield (BAML HY) corporate bonds (after option adjustments), in red, offered promised spreads of about 400-600 bp, higher in recessions, and a spike as large as 2,000 bp (!) in the Great Recession. Source: FRED (Moody's and Bank of America Merrill Lynch).

Credit Default Swaps

A large newish market: credit default swaps.

The financial world is always changing and innovating. The components of bond returns described above used to be primarily a conceptual curiosity—firms would borrow money from their lenders, paying one interest rate that just contained all premiums. But then, with the introduction of **credit default swaps** (often abbreviated **credit swaps** or **CDS**), some premium components suddenly became themselves tradable.

A CDS example: The swap seller insures the swap buyer.

Here is an example of a CDS: A large pension fund that owned a $15 million bond issued by *Hospital Corporation of America* (HCA) may have wanted to purchase a $10 million credit swap from a hedge fund that in turn wanted to bet that HCA would *not* go bankrupt. Upon HCA bankruptcy, the hedge fund would owe the pension fund $10 million. The *Wall Street Journal* reported that this CDS contract cost about $130,000 in June 2006, but rose to over $400,000 in July, because of a potential buyout deal that increased the risk of future default. And, in this case, purchasing the CDS in June would have been a lucky deal for the pension fund and an unlucky deal for the hedge fund, because HCA did indeed go bankrupt.

Conceptually...

The best way to think of such credit swaps is as insurance contracts, in which the swap sellers (the hedge funds) are the insurance providers. The seller of the swap thus takes on the credit risk from the buyer—just like an insurance company takes on some risk from the insured party—in exchange for a premium payment upfront. The insurance then usually pays out in case of a credit event (e.g., a failed payment or bankruptcy)—typically for one particular bond within a given number of years. The payment itself can be formula-determined, or it can be a guarantee by the CDS seller to buy the bond at a predetermined price. One way of thinking of the upfront cost (the $130,000 that increased to $400,000) is that it contains the bond's default premium.

In effect, credit swaps allow investors to hold different premium components.

Credit swaps allow different funds to hold different premiums of a bond. In our example, the pension fund decided to earn primarily the time premium component of HCA's bonds, divesting itself of the credit risk and other components. The hedge fund took over the credit premium. It decided to speculate that HCA would not go bankrupt, and it could do so without having to take a large cash position in HCA's bonds. Of course, hedge funds and other investors could also have speculated with CDS's that HCA would go bankrupt.

The CDS market size was huge.

➤ Over-the-counter, Sect. 7.2, Pg.159.

Credit swaps are typically traded in lots of $5 million and last for 5 years (but 3 to 10 years is not unusual, either). This market is **over-the-counter** (**OTC**)—that is, negotiated one-to-one between two parties. This market is also very big: in 2016, there was more than $17 trillion outstanding in single-name swaps.

The financial crisis and the CDS collapse. The shifting of risk everywhere.

The CDS market collapsed temporarily in the financial crisis of 2008. An important insurer, American International Group (AIG), had a financial arm that had sold too many CDSs and the traders booked them as outright profit. This worked for a while...until it did not. We, the taxpayers, had to bail out AIG, because the Treasury feared that too many bond buyers were relying on AIG's insurance, and would themselves have to default if this insurance had become worthless. Unfortunately, even after the market has risen again (after 2010), it still remains relatively "dark": no one really knows how big or small it is, who is trading, who has exposures, and so on—and this includes the Federal Reserve and the Treasury. In 2007, I wrote in the first edition of this book that no one knows who is really holding most of the credit risk in the economy nowadays. I gave as an example the German bank IKB, which had collapsed to everyone's surprise, because it had owned too many financial securities that were tied to U.S. mortgages. I was either prescient or just lucky. During the 2008 financial crisis (the Great Recession), investors did not want to trust even good corporations and banks any longer, simply because they did not know what their actual exposures were. The CDS market is large and competitive—but also opaque and rife with manipulation. Some corporations have even been pressured into defaulting for the sake of triggering CDS. And, in the believe-it-or-not category, there is actually a committee of (conflicted) major banks that can decide whether a bond is really in default or not when the creditor offers an exchange. However, CDSs are not intrinsically evil. Like most other financial instruments, they can be used to reduce or increase risk. The social problem, even today, is that (1) traders have incentives to speculate too much with them, because their risk is hard to measure, and banks and their shareholders intrinsically like risk (but in cases of a crisis, it is taxpayers that will be on the hook again!); and (2) no one really knows what is going on in this OTC market (and many other financial markets). If large institutions speculate too much with them (and pay non-recoverable bonuses based on estimated profitability) and then fail, we the people may have no choice but to bail them out *again*. Despite these problems, the CDS market has come back strongly. As of 2016, there is "only" about $500-$600 trillion in notional amounts outstanding, with $15-$20 trillion in gross market value. By any measure, these numbers are *huge*.

6.3 Uncertainty in Capital Budgeting

Let's now return to the basic tasks of capital budgeting: selecting projects under uncertainty. Your task is to compute present values with imperfect knowledge about future outcomes. The principal tool in this task will be the **payoff table** (or **state table**), which assigns probabilities to the project value in each possible future-value-relevant scenario. For example, the value of a factory producing hard disks may depend on computer sales (say, low, medium, or high), whether hard disks have become obsolete (yes or no), whether the economy is in a recession or expansion, and what the oil price (a major transportation cost factor) turns out to be. It is the manager's task to create the appropriate "state" table, which specifies what variables and scenarios are most value-relevant and how the business will perform in each of them. Clearly, it is not an easy task even to understand what the key factors are, much less to determine the probabilities under which these factors will take on one or another value. Assessing how your own project will respond to them is an even harder task—but it is an inevitable one. If you want to understand the value of your project, you must understand what your project's key value drivers are and how your project will respond to these value drivers. Fortunately, for many projects, it is usually not necessary to describe all possible outcomes in the most minute detail—just a dozen or so scenarios are often enough to cover the most important possibilities.

Next you learn about payoff diagrams, to characterize the main future contingencies.

Present Value with Outcome-Contingent Payoff Tables

We begin with the hypothetical purchase of a building for which the future value is uncertain. Next year, this investment will be worth either \$60 thousand (with 1/4 probability) or \$100 thousand (with 3/4 probability). (In case you are worried that real firms last longer than one year, you can think of these values as themselves reflecting further future outcomes for the firm.) To help you remember the two possible states, let's just call the bad outcome "Rain" and the good outcome "Sun." (If you are from California, be aware that rain is the bad outcome and sun is the good outcome.)

Our example of this section: A building can end up with one of two possible future values.

The Building's Expected Value

If you own the full building, your payoff table, omitting thousands henceforth, is as follows:

A payoff table.

Event	Probability	Value
Rain	1/4	\$60
Sun	3/4	\$100
⟹ **Expected Future Value**		\$90

The expected future building value of \$90 (thousand) was computed as

$$\mathsf{E}\big(\text{Value at Time 1}\big) = 1/4 \cdot \$60 + 3/4 \cdot \$100 = \$90$$
$$= \text{Prob} \cdot \text{Value Rain} + \text{Prob} \cdot \text{Value Sun}$$

To obtain the expected future cash value of the building, multiply each possible outcome by its probability.

This is *not* yet discounted. It is only your expectation of the future outcome.

Now assume that the appropriate expected rate of return for a project of type "building" with this type of riskiness and with 1-year maturity is 20%. (This 20% discount rate is provided by demand and supply in the financial markets, and it is assumed to be known by you, the manager.) Your goal is to determine the present value—the appropriate price—for the building *today*.

Then discount back the expected cash value using the appropriate cost of capital.

There are two methods to arrive at the present value of the building—and they are almost identical to what you have done earlier. You only need to replace the known value with the expected value, and the known future rate of return with an expected rate of return. The first PV

You can use NPV with expected (rather than actual, known) cash flows and expected (rather than actual, known) rates of return.

method is to compute the expected value of the building next period and to discount it at the cost of capital, here 20%:

$$\text{PV} = \frac{\$90}{1 + 20\%} \approx \$75$$
$$= \frac{\mathcal{E}(\text{Value at Time 1})}{1 + \mathcal{E}(r)}$$

Taking expectations and discounting can be done in any order.

The second method is to compute the discounted state-contingent value of the building, and then take expected values. To do this, augment the earlier table:

Event	Probability	Value	Discount Factor	⇒	PV
Rain	$1/4$	\$60	1/(1+20%)	⇒	\$50
Sun	$3/4$	\$100	1/(1+20%)	⇒	\$83.33

If it rains, the present value is \$50. *If* the sun shines, the present value is \$83.33. Thus, the expected value of the building can also be computed as

$$\mathcal{E}(\text{Value at Time 1}) = 1/4 \cdot \$50 + 3/4 \cdot \$83.33 = \$75$$
$$= \text{Prob} \cdot \text{Value if Rain} + \text{Prob} \cdot \text{Value if Sun}$$

Both methods lead to the same result: You can either first compute the expected value of the investment next year ($1/4 \cdot \$60 + 3/4 \cdot \$100 = \$90$) and then discount this expected value of \$90 to \$75; or you can first discount all possible future outcomes (\$60 to \$50, and \$100 to \$83.33) and then compute the expected value of the discounted values ($1/4 \cdot \$50 + 3/4 \cdot \$83.33 = \$75$.)

IMPORTANT

Under uncertainty, in the NPV formula,

- known future cash flows are replaced by expected discounted cash flows, and
- known appropriate rates of return are replaced by appropriate expected rates of return.

You can first do the discounting and then take expectations, or vice-versa. The order does not matter.

The State-Contingent Rates of Return

The state-contingent rates of return can also be probability-weighted to arrive at the average (expected) rate of return.

What would the rates of return be in the two states, and what would your overall expected rate of return be? If you have bought the building for \$75 and it will be sunny, your actual rate of return will be

$$\text{If Sun:} \quad r \approx \frac{\$100 - \$75}{\$75} \approx +33\%$$

If it's rainy, your rate of return will be

$$\text{If Rain:} \quad r \approx \frac{\$60 - \$75}{\$75} \approx -20\%$$

Therefore, your expected rate of return is

$$\mathcal{E}(r) \approx 1/4 \cdot (-20\%) + 3/4 \cdot (+33\%) \approx 20\%$$
$$\text{Prob} \cdot \text{Rain Rate of Return} \quad \text{Prob} \cdot \text{Sun Rate of Return}$$

The probability state-weighted rates of return add up to the expected overall rate of return. This is as it should be: After all, you derived the proper price of the building today using a 20% expected rate of return.

Q 6.12. What changes have to be made to the NPV formula to handle an uncertain future?

Q 6.13. A factory can be worth $500,000 or $1,000,000 in two years, depending on product demand, each with equal probability. The appropriate cost of capital is 6% per year. What is the present value of the factory?

Q 6.14. A new product may be a dud (20% probability), an average seller (70% probability), or dynamite (10% probability). If it is a dud, the payoff will be $20,000; if it is an average seller, the payoff will be $40,000; and if it is dynamite, the payoff will be $80,000.

1. What is the expected payoff of the project?
2. The appropriate expected rate of return for such payoffs is 8%. What is the PV of the payoff?
3. If the project is bought for the appropriate present value, what will be the rates of return in each of the three outcomes?
4. Confirm the expected rate of return when computed from the individual outcome-specific rates of return.

6.4 Splitting Uncertain Project Payoffs into Debt and Equity

Most projects are financed with a mix of debt and equity.

The most important reason for you to learn about state payoff tables is that they will help you understand cash flow rights. This leads to one of the most important concepts in finance: the difference between a **loan** (also called **debt** or **leverage**) and **levered ownership** (also called **levered equity** or simply **equity** or **stock**). Almost all companies and projects are financed with both debt and levered equity. You already know in principle what debt is. Levered equity is simply what accrues to the business owner *after* the debt is paid off. We leave it to later chapters to make a distinction between financial debt and other obligations—for example, tax obligations—and to cover the control rights that flow from securities—for example, how debt can force borrowers to pay up and how equity can replace poorly performing managers.

Other projects are financed the same way.

You probably already have an intuitive understanding about the distinction between debt and equity. If you own a house with a mortgage, you really own the house only after you have made all debt payments. If you have student loans, you *yourself* are the levered owner of your future income stream. That is, you get to consume "your" residual income only *after* your liabilities (including your nonfinancial debt) are paid back. But what will the levered owner and the lender get if the company's projects fail, if the house collapses, or if your career takes a turn toward Rikers Island? What is the appropriate compensation for the lender and the levered owner? The split of net present value streams into loans (debt) and levered equity lies at the heart of finance.

Outcome (or "state")-contingent claims have payoffs that depend on future states of nature.

You now know how to compute the present value of state-contingent payoffs—your building paid off differently in two different states of nature. Thus, your building was a state-contingent claim—its payoff depended on the outcome. But it is just one of many possible state-contingent claims. Another might promise to pay $1 if the sun shines and $25 if rain falls. Using payoff tables, you can work out the value of *any* state-contingent claim and, in particular, the value of our two most important state-contingent claims, debt and equity.

The Loan

Assume that the building is funded by (a) a mortgagor and (b) a residual (the levered building owner).

Let's assume you want to finance the building purchase of $75 with a mortgage of $70. In effect, the single project "building" is being turned into two different projects, each of which can be owned by a different party. The first project is "Mortgage Lending." The second project is "Residual Building Ownership," that is, ownership of the building but bundled with the obligation to repay the mortgage. The "Residual Building Ownership" investor will not receive a dime until *after* the debt has been satisfied. As already explained, such residual ownership is called levered equity, or just equity (or even stock) in the building. This avoids calling it "what's-left-over-after-the-loans-have-been-paid-off."

The first goal is to determine the appropriate promised interest rate on a "$70 value today" mortgage loan on the building.

What sort of interest rate would the creditor demand? To answer this question, you need to know what will happen if the building were to be worth less than than the mortgage promise. Let's say that the value of the building will be $60 next year if rain falls. (The roof is partly water-soluble.) We are assuming that the owner could walk away, and the creditor could repossess the building but not any of the borrower's other assets. Such a mortgage loan is called a **no-recourse loan**. There is no recourse other than taking possession of the asset itself. This arrangement is called **limited liability**. The building owner cannot lose more than the money that he originally puts in. Limited liability is the mainstay of many financial securities: For example, if you buy stock in a company in the stock market, you cannot be held liable for more than your investment, regardless of how badly the company performs.

Start with the payoff table, and write down payoffs to project "Mortgage Lending."

➤ Credit Risk, Sect. 6.2, Pg.110.

To compute the present value for the project "Mortgage Lending," return to the problem of setting an appropriate interest rate, given credit risk (from Section 6.2). Start with the following payoff table:

Event	Prob	Value	Discount Factor
Rain	1/4	$60	1/1.20
Sun	3/4	Promised	1/1.20

Limited Liability

Limited liability was invented after the Renaissance, but it became common only in the nineteenth and twentieth centuries. Ultimately, it is this legal construction that allowed corporations to evolve into entities distinct from their owners. Thus, in 1911, the President of Columbia University wrote: "The limited liability corporation is the greatest single discovery of modern times.... Even steam and electricity are less important."

William Goetzmann, Yale University

The quoted (or promised) payoff.

The creditor receives the property worth $60 if it rains, or the full promised amount (to be determined) if the sun shines. To break even, the creditor must solve for the payoff to be received if the sun shines in exchange for lending $70 today. This is the "quoted" or "promised" payoff:

$$\$70 = 1/4 \cdot \left(\frac{\$60}{1+20\%}\right) + 3/4 \cdot \left(\frac{\text{Promise}}{1+20\%}\right)$$

$$\text{Loan Value}_0 = \text{Prob} \cdot \text{Rain Loan PV} + \text{Prob} \cdot \text{Sun Loan PV}$$

You can solve this for the necessary promise, which is

Nerdnote: Special liability and tax rules apply to private residences. Mortgages can have limited liability ("non recourse") or unlimited liability ("full recourse"). The latter can also have further nasty tax consequences, where a capital loss in the home can create a large ordinary income tax obligation, adding insult to injury. (If interested, google for "cancellation-of-debt income.") Moreover, as a home owner, you can deduct interest only on the first $1 million in mortgage; and capital losses on the home do not create a tax credit, but large capital gains can create a tax obligation.

$$\text{Promise} = \frac{(1+20\%)\cdot \$70 - {}^1\!/_4 \cdot \$60}{{}^3\!/_4} = \$92$$

$$= \frac{[1+\mathsf{E}(r)]\cdot \text{Loan Value}_0 - \mathsf{Prob}(\text{Rain})\cdot \text{Rain Value}}{\mathsf{Prob}(\text{Sun})}$$

in repayment, paid by the borrower only if the sun shines.

The state-contingent rates of return in the rainy ("default") state and in the sunny ("solvent") state can be probability-weighted to arrive at the expected rate of return.

With this promised payoff of \$92 (if the sun shines), the lender's rate of return will be the **promised rate of return**:

$$\text{If Sun:} \quad r = \frac{\$92 - \$70}{\$70} \approx +31.4\%$$

The lender would not provide the mortgage at any lower promised interest rate. If it rains, the owner walks away, and the lender's rate of return will be

$$\text{If Rain:} \quad r = \frac{\$60 - \$70}{\$70} \approx -14.3\%$$

Therefore, the lender's *expected* rate of return is

$$\mathsf{E}(r) = \underbrace{{}^1\!/_4}_{\mathsf{Prob}} \cdot \underbrace{(-14.3\%)}_{\text{Rain Rate of Return}} + \underbrace{{}^3\!/_4}_{\mathsf{Prob}} \cdot \underbrace{(+31.4\%)}_{\text{Sun Rate of Return}} \approx 20\%$$

The stated rate of return is 31.4% (and it is not an exorbitant rate!), but the expected rate of return is 20%. After all, in our risk-neutral perfect market, anyone investing for one year expects to earn an expected rate of return of 20%.

The Levered Equity

Now compute the payoffs of the post-mortgage (i.e., levered) ownership of the building. The method is exactly the same.

As the residual building owner, what rate of return would you expect as proper compensation? You already know the building is worth \$75 today. Thus, after the loan of \$70, you need to pay in \$5—presumably from your personal savings. Of course, you must compensate your lender: To contribute the \$70 to the building purchase today, you must promise to pay the lender \$92 next year. If it rains, the lender will confiscate your house, and all your invested personal savings will be lost. However, if the sun shines, the building will be worth \$100 minus the promised \$92, or \$8. Your payoff table as the levered equity building owner is as follows:

Event	Prob	Value	Discount Factor
Rain	1/4	\$0	1/1.20
Sun	3/4	\$8	1/1.20

This allows you to determine that the *expected* future levered building ownership payoff is ${}^1\!/_4 \cdot \$0 + {}^3\!/_4 \cdot \$8 = \$6$. Therefore, the present value of levered building ownership is

$$\text{PV} = \underbrace{{}^1\!/_4}_{\mathsf{Prob}} \cdot \underbrace{\left(\frac{\$0}{1+20\%}\right)}_{\text{Rain PV}} + \underbrace{{}^3\!/_4}_{\mathsf{Prob}} \cdot \underbrace{\left(\frac{\$8}{1+20\%}\right)}_{\text{Sun PV}} \approx \$5$$

Your rates of return are

Again, knowing the state-contingent cash flows permits computation of state-contingent rates of return and the expected rate of return.

$$\text{If Sun:} \quad r \approx \frac{\$8 - \$5}{\$5} = +60\%$$

$$\text{If Rain:} \quad r \approx \frac{\$0 - \$5}{\$5} = -100.00\%$$

The expected rate of return of levered equity ownership, that is, the building with the bundled mortgage obligation, is

$$\mathcal{E}(r) = \underbrace{1/4}_{\text{Prob}} \cdot \underbrace{(-100.00\%)}_{\text{Rain Rate of Return}} + \underbrace{3/4}_{\text{Prob}} \cdot \underbrace{(+60\%)}_{\text{Sun Rate of Return}} = 20\%$$

Reflections On The Example: Payoff Tables

Payoff tables are great conceptual tools.

Payoff tables are fundamental tools to help you think about projects and financial claims. Admittedly, they can sometimes be tedious, especially if there are many different possible states. (There may even be infinitely many states, as in a bell-shaped, normally-distributed project outcome—but you can usually approximate even the most continuous and complex outcomes fairly well with no more than 10 discrete possible outcomes.)

There are three possible investment opportunities here. The bank is just another investor, with particular payoff patterns.

Exhibit 6.6 shows how elegant such a table can be. It describes everything you need in a very concise manner: the state-contingent payoffs, expected payoffs, net present value, and expected rates of return for your house scenario. Because owning the mortgage and the levered equity is the same as owning the full building, the last two columns must add up to the values in the "Building Value" column. You could decide to be any kind of investor: a creditor (bank) who is loaning money in exchange for promised payment; a levered building owner who is taking a "piece left over after a loan"; or an unlevered building owner who is investing money into an unlevered project (i.e., the whole piece). All three investments are just state-contingent claims.

Event	Prob	Building Value	$92-Promise Mortgage	Levered Equity
Rain	1/4	$60	$60	$0
Sun	3/4	$100	$92	$8
Expected Value at Time 1		$90	$84	$6
Present Value at Time 0		$75	$70	$5
From Time 0 to Time 1, $\mathcal{E}(r)$		20%	20%	20%

Exhibit 6.6: *Payoff Table and Overall Values and Returns.* In this example, the project is financed with $70 in mortgage promising $92 in payment.

IMPORTANT

Whenever possible, in the presence of uncertainty, write down a payoff table to describe the probabilities of each possible event ("state") with its state-contingent payoff.

Q 6.15. In the example, the building was worth $75, the mortgage was worth $70, and the equity was worth $5. The mortgage thus financed about 93.3% of the cost of the building, and the equity financed 6.7%. Is the arrangement identical to one in which two partners purchase the building together—one puts in $70 and owns 93.3% of the building, and the other puts in $5 and owns 6.7%?

Q 6.16. Buildings are frequently financed with mortgages that cover 80% of the purchase price, not 93.3% ($70 of $75). Produce a table similar to Exhibit 6.6 for this case.

Ivo Welch, Corporate Finance (4th Edition), July 2017.

Reflections on The Example: Debt and Equity Risk

We have only briefly mentioned risk. It was just not necessary to illustrate the main insights. In a risk-neutral world, all that matters is the expected rate of return, not the uncertainty about what you will receive. Of course, you can assess the risk even in our risk-neutral world where risk earns no extra compensation (a risk premium). So, which investment is riskiest: full ownership, loan ownership, or levered ownership?

Evaluate the risk of the three types of projects, even if riskier projects do not earn higher expected rates of return.

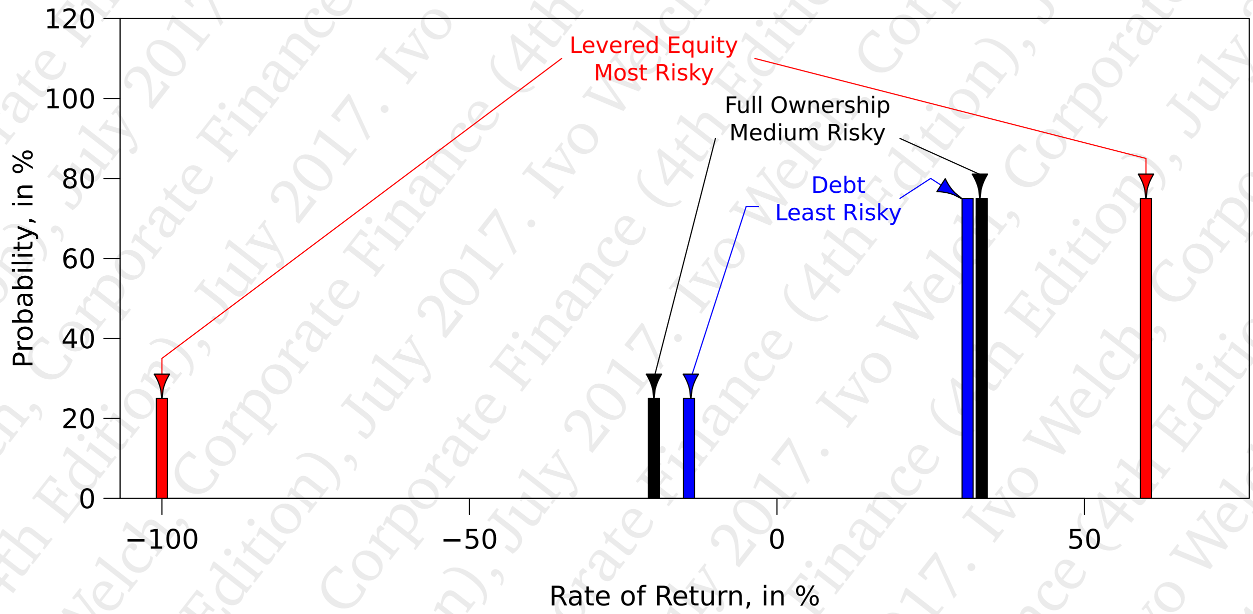

Exhibit 6.7: *Three Probability Histograms for Project Rates of Return.* The solid red bars are the payoffs to equity, the riskiest investment. The solid black bars are the payoffs to full ownership. The blue bars are the payoffs to debt, the least risky investment. You can judge risk by how spread out the two bars are.

Exhibit 6.7 plots the histograms of the rates of return for each of the three types of investments. The equity loses everything (–100%) with a $1/4$ probability but earns 60% with $3/4$ probability. The debt loses about 14.3% with $1/4$ probability and gains 31.4% with $3/4$ probability. The full ownership loses about –20% with $1/4$ and gains 33.3% with $3/4$ probabilities. As the visuals show, the loan is least risky, followed by the full ownership, followed by the levered ownership. There is an interesting intuition here. By taking the mortgage, the medium-risk project "building" has been split into one more risky project ("levered building") and one less risky project ("mortgage"). The combined "full building ownership" project therefore has an average risk.

Leveraging (mortgaging) a project splits it into a safer loan and a riskier levered ownership.

Of course, regardless of leverage, all investment projects in our risk-neutral world expect to earn a 20% rate of return. After all, 20% is the universal time premium here for investing money. (The default premium is a component only of promised interest rates, not of expected interest rates; see Section 6.2.) By assuming that investors are risk-neutral, we have assumed that the risk premium is zero. Investors are willing to take any investment that offers an expected rate of return of 20%, regardless of risk. (If investors were risk-averse, debt would offer a lower expected rate of return than the project, which would offer a lower expected rate of return than equity.)

If everyone is risk-neutral, everyone should expect to earn 20%.

Unrealistic, maybe! But ultimately, this is the basis for more realistic examples, and illustrative of the most important concepts.

Although our example was a little sterile because we assumed away risk preferences, it is nevertheless very useful. Almost all projects in the real world are financed with loans extended by one party and levered ownership held by another party. Understanding debt and equity is as important to corporations as it is to building owners. After all, stocks in corporations are basically levered ownership claims that provide money only *after* the corporation has paid back its liabilities. The building example has given you the skills to compute state-contingent, promised, and expected payoffs, as well as state-contingent, promised, and expected rates of return. These are the necessary tools to work with debt, equity, or any other state-contingent claim. And really, all that will happen later when we introduce risk aversion is that you will add a few extra basis points of required compensation—more to equity (the riskiest claim), fewer to the project (the medium-risk claim), and still fewer to debt (the safest claim).

Q 6.17. Compare a "junk" mortgage (with its requisite junk equity, receiving payments only if the junk mortgage is paid off) that promises to pay off $70 with a "solid" mortgage (with its requisite solid equity) that promises to pay off $60.

1. Does the junk mortgage seem riskier than the solid mortgage?
2. Does the junk equity seem riskier than the solid equity?
3. Does the building seem riskier if financed with a junk mortgage rather than with a solid mortgage?

What "Leverage" Really Means—Financial and Operational Leverage

Leverage "amplifies" the equity stake.

I have already mentioned that debt is often called leverage and equity is called "levered equity." Let me now explain why. A lever is a mechanical device that can amplify effects. In finance, a lever is something that allows a smaller equity investment to still control the firm and be more exposed to the underlying firm's gain or loss than unlevered ownership. That is, with leverage, a small change in the underlying project value translates into a larger change in value for levered equity, both up and down. You have seen this leverage mechanism in our house example above, and specifically in Exhibit 6.7. Ordinary ownership would have cost you $75. But with leverage, you could take control of the house with cash of only $5. In addition, it also meant that if the sun had shone, you would have earned ($8 – $5)/$5 = 60%, not just ($100 – $75)/$75 = 33%; but if it had rained, you would have earned –100% (lost *everything*), not just ($60 – $75)/$75 = –20%. Leverage amplified your stake.

The leverage concept can encompass more than just financial debt.

➤ Calculating leverage, Sect. 15.6, Pg.420.

Financial debt is a lever, but it is not the only one. Leverage can also be calculated using all corporate liabilities (which may include, e.g., accounts payable and pension obligations). More importantly, because leverage is a general concept rather than an accounting term, you should think of it in even broader terms. The idea of leverage is always that a smaller equity investment can control the firm and is more sensitive to firm value changes. Exhibit 6.8 illustrates some different types of levers. In this table, you can pay $475 for machine and labor, and receive either $200 or $1,000 in product revenues, plus $150 as resale value for the machine. In the first line of the exhibit, you can see that the bad state, you lose 26%; and in the good state, you earn 142%. The second line shows that financial leverage can magnify these rates of return into –100% or +540%. But instead of taking on financial debt, you could also lease the machine, which costs you $250 in leasing fees (with no residual ownership of the machine at the end), and pay for labor of $75. In this case, you have effectively levered up, increasing your risk to –38% and +208% but without taking on any financial leverage. It is the lease that has now become your leverage! And you can also combine real and financial leverage. Finally, there can even be differences in the degree to which the production technologies themselves are levered. The final example in the fourth line shows a different method of production, which is intrinsically more levered.

Example Assumptions:

- The machine costs $400 and can be resold for $150. The net operating cost is thus $250.
- In a hypothetical lease, the lessee would pay the lessor $250 for use of the machine, and the lessor would own the machine ($150) at the end.
- Labor costs are $75.
- Product produces $200 ("Bad") or $1,000 ("Good").
- Assume prevailing interest rate is 0.

Leverage	Investment	Out of Pocket	Dollars Bad	Dollars Good	Percent Bad	Percent Good	FLR
None	Pay for everything.	$475	$350	$1,150	–26%	+142%	0%
Financial	Borrow $350.	$125	$0	$800	–100%	+540%	74%
Real	Lease machine, Pay $250.	$325	$200	$1,000	–38%	+208%	0%
Real+Financial	Lease machine & Borrow $200.	$125	$0	$800	–100%	+540%	62%
Different Technology—Labor costs $40, different machine costs $400, has residual value of $115.							
Technology	Pay for everything.	$440	$315	$1,115	–28%	+153%	0%

Exhibit 6.8: *Financial and Real Leverage.* FLR is financial leverage, which is defined as the fraction of financial debt divided by the sum of debt and equity.

Working with More Than Two Possible Outcomes

Multiple outcomes will cause multiple breakpoints in the relation from promised to expected payoffs.

In the real world, possible outcomes can often range from 0 to infinity. Can you use the same method if you have more than two scenarios? For example, assume that the building could be worth $60, $70, $80, $90, or $100 with equal probability (for an expected value of $80) and that the appropriate expected interest rate is 20%. It follows that the building has a PV of $80/1.20 ≈ $67. If a loan promised to pay $60 at time 1, how much would it expect to receive? The full $60, of course, because the building is always worth at least this much:

$$\mathrm{E}\Big(\text{Payoff}(\ \$0 \le \ \text{Loan Promise} = x \ \le \$60)\ \Big) \ = 100\% \cdot x$$

But if a loan promised $61, how much would it expect to receive then? It would expect $60 for sure, plus the extra "marginal" $1 with 80% probability (because there is an 80% chance that $61 is covered; only if the outcome is $60, which happens 20% of the time, would it not receive the full $61). Thus, for the $61 loan promise, it would expect to receive $60.80. In fact, it would expect only 80 cents for each dollar promised between $60 and $70. So, if a loan promised x between $60 and $70, it would expect to receive

$$\mathrm{E}\Big(\text{Payoff}(\ \$60 \le \ \text{Loan Promise} = x \ \le \$70\ )\ \Big) \ = \$60 + 80\% \cdot (x - \$60)$$

If a loan promised $71, how much would it expect to receive? It would expect $60 for sure, plus $8 for the next promised $10, plus 60 cents on the dollar for anything above $70, i.e., $68.60,

$$\mathrm{E}\Big(\text{Payoff}(\ \$70 \le \ \text{Loan Promise} = x \ \le \$80\ )\ \Big) \ = \$60 + \$8 + 60\% \cdot (x - \$70)$$

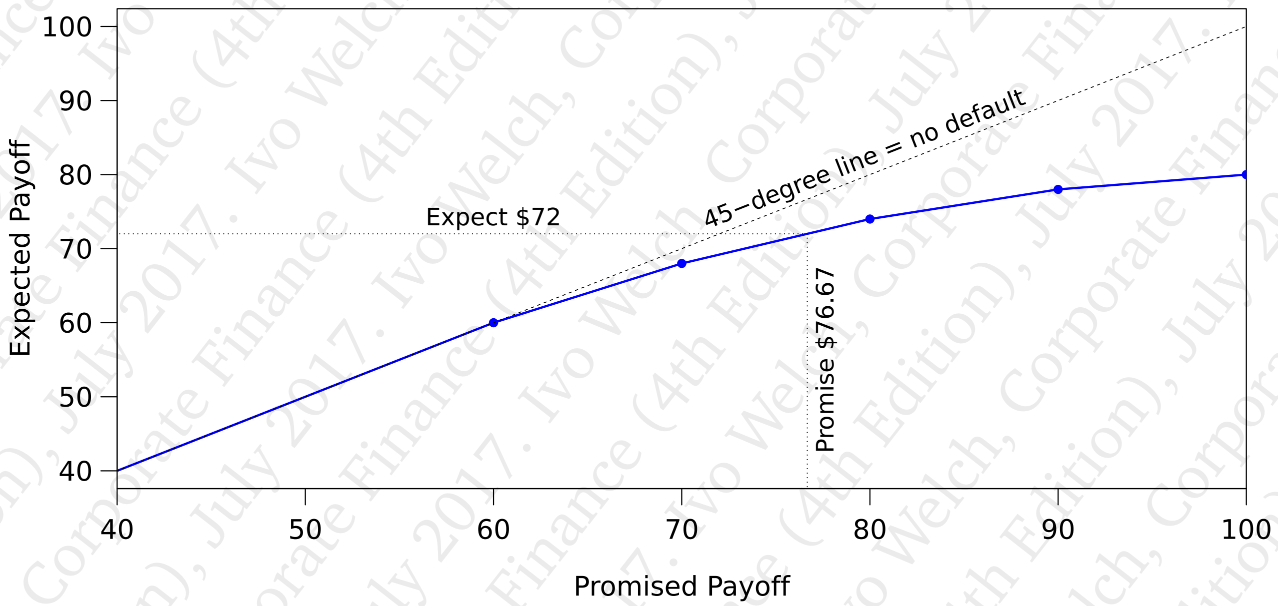

Exhibit 6.9: *Promised versus Expected Payoff for a Loan on the Project with Five Possible Payoffs.* The dotted line is one-to-one, where a promised dollar is an expected dollar (i.e., risk-free). The blue line shows the payoffs to the bond. The firm will be worth \$60, \$70, \$80, \$90, or \$100, each with equal probability. To borrow \$60 today, the bond must offer an expected payoff of $\$60 \cdot 1.2 = \$72$ next year. Following the arrow from the y-axis at \$72 to the payoff function and then down to the x-axis shows that this high an expected payoff requires a promised payoff of \$76.67.

You can now read off the appropriate promised value from the graph for any mortgage.

Exhibit 6.9 plots these expected payoffs as a function of the promised payoffs. With this figure, mortgage valuation becomes easy. For example, how much would the loan have to promise to provide \$50 today? The expected payoff would have to be $(1 + 20\%) \cdot \$50 = \$60$. This is on the linear segment, so you would have to promise \$60. Of course, you cannot offer an expected payoff of more than \$80, so forget about borrowing more than $\$80/1.2 \approx \$66.67$ today.

The same approach would also work when possible value outcomes are "normally distributed" (i.e., following a bell-curve). The math is a more complex, but the method remains the same.

Q 6.18. What is the formula for a promised loan payoff between \$80 and \$90?

Q 6.19. What is the expected payoff if the promised payoff is \$72?

Q 6.20. If you want to borrow \$65, what do you have to promise?

Q 6.21. If there were infinitely many possible outcomes (e.g., if the building value followed a statistical normal distribution), what would the graph of expected payoffs of the loan as a function of promised payoffs look like?

Q 6.22. A new product may be a dud (20% probability), an average seller (70% probability), or dynamite (10% probability). If it is a dud, the payoff will be \$20,000; if it is an average seller, the payoff will be \$40,000; if it is dynamite, the payoff will be \$80,000. The appropriate expected rate of return is 6% per year. If a loan promises to pay off \$40,000, what are the promised and expected rates of return?

An Error: Discounting Promised Cash Flows with the Promised Cost of Capital

A common mistake is the attempt to avoid the need to estimate expected values by discounting promised cash flows with promised discount rates. After all, both numbers reflect default risk. The two default issues might cancel out one another, and you might end up with the correct inference. *Or they might not cancel out, in which case you will end up with an incorrect decision!*

To illustrate, say the appropriate expected rate of return is 20%. A newly available bond investment promises $25 for a $100 investment with fully insured principal but a 50% probability of default on the interest payment. Say that other risky bonds in the economy offer 11.4% (for a total interest rate of 31.4%). If you discounted the promised interest payment of $25 with the quoted interest rate on your benchmark bonds, you would get

$$\text{Bad NPV Calculation} = -\$100 + \frac{\$100}{1+20\%} + \frac{\$25}{1+31.4\%} \approx +\$2.36$$

Wrong! Instead, you must work with expected values:

$$\text{Correct NPV Calculation} = -\$100 + \frac{\$100}{1+20\%} + \frac{\$12.50}{1+20\%} = -\$6.25$$

This bond would be a bad investment.

Summary

This chapter covered the following major points:

- Uncertainty means that a project may not return its promised amount.
- A random variable is one whose outcome has not yet been determined. It is characterized by its distribution of possible future outcomes.
- The "expected value" is the probability-weighted sum of all possible outcomes. It is the "average" or "mean," but it is applied to the future instead of to a historical data series. It is a measure of "reward."
- Risk neutrality means indifference between a safe bet and a risky bet if their expected rates of return are the same.
- The possibility of future default causes promised (quoted) interest rates to be higher than expected interest rates. Default risk is also often called credit risk.
- Most of the difference between promised and expected interest rates is due to default. Extra compensation for bearing more risk—the risk premium—and other premiums are typically smaller than the default premium for bonds.
- Credit ratings can help judge the probability of potential losses in default. Moody's and S&P are the two most prominent vendors of ratings for corporate bonds.
- The key tool for thinking about uncertainty is the payoff table. Each row represents one possible outcome, which contains the probability that the state will come about, the total project value that can be distributed, and the allocation of this total project value to different state-contingent claims. The state-contingent claims "carve up" the possible project payoffs.
- Most real-world projects are financed with the two most common state-contingent claims—debt and equity. Their payoff rights are best thought of in terms of payoff tables.
- Debt and equity are methods to parcel out total firm risk into one component that is safer than the overall firm (debt) and one that is riskier than the overall firm (equity).
- The presence of debt "levers up" equity investments. That is, a smaller upfront cash investment becomes more exposed to swings in the value of the underlying firm. However, there are also other leverage mechanisms that firms can choose (e.g., leasing or technology).
- If debt promises to pay more than the project can deliver in the worst state of nature, then the debt is risky and requires a promised interest rate in excess of its expected interest rate.
- NPV is robust to modest errors in the expected interest rate (the discount rate) for near-term cash flows. However, NPV is not necessarily robust with respect to modest errors in either expected cash flows or discount rates for distant cash flows.
- NPV is about discounting *expected* cash flows with *expected* rates of return. You cannot discount *promised* cash flows with *promised* rates of return.

Keywords

Answers

Q 6.1 No! The expected outcome (value) is assumed to be known—at least for an untampered die throw. The following is almost philosophy and beyond what you are supposed to know or answer here: It might, however, be that the expected value of an investment is not really known. In this case, it, too, could be a random variable in one sense—although you are assumed to be able to form an expectation (opinion) over anything, so in this sense, it would not be a random variable, either.

Q 6.2 If you do not know the exact bet, you may not know the expected value, which means that even the expected value is unknown. This may be the case for stocks, where you are often forced to guess what the expected rate of return will be (unlike for a die, for which you know the underlying physical process, which assures an expected value of 3.5). However, almost all finance theories assume that you know the expected value. Fortunately, even if you do not know the expected value, finance theories hope you still often have a pretty good idea.

Q 6.3 If the random variable is the number of dots on the die times two, then the expected outcome is $1/6 \cdot (2) + 1/6 \cdot (4) + 1/6 \cdot (6) + 1/6 \cdot (8) + 1/6 \cdot (10) + 1/6 \cdot (12) = 7$. The realization was 12.

Q 6.4 The expected value of the stock investment is $5\% \cdot (\$41) + 10\% \cdot (\$42) + 20\% \cdot (\$45) + 30\% \cdot (\$48) + 20\% \cdot (\$58) + 10\% \cdot (\$70) + 5\% \cdot (\$75) = \$52$. Therefore, buying the stock at \$50 is not a fair bet, but it is a good bet.

Q 6.5 The variance of the P_{+1} stock investment is $\mathsf{Var}\left(P_{+1}\right) = 5\% \cdot (\$41 - \$52)^2 + 10\% \cdot (\$42 - \$52)^2 + 20\% \cdot (\$45 - \$52)^2 + 30\% \cdot (\$48 - \$52)^2 + 20\% \cdot (\$58 - \$52)^2 + 10\% \cdot (\$70 - \$52)^2 + 5\% \cdot (\$75 - \$52)^2 = 5\% \cdot \$\$121 + 10\% \cdot \$\$100 + 20\% \cdot \$\$49 + 30\% \cdot \$\$16 + 20\% \cdot \$\$36 + 10\% \cdot \$\$324 + 5\% \cdot \$\$529 = \underline{\$\$96.70}$. Therefore, the standard deviation (risk) is $\mathsf{Sdv}\left(P_{+1}\right) = \sqrt{\$\$96.70} \approx \$9.83$.

Q 6.6 Investors are more risk-averse for large bets relative to their wealth.

Q 6.7 Yes, individual investors are typically more risk-averse than investors in the aggregate. This can even be the case for all investors.

Q 6.8 Expected and promised rates are the same only for risk-free (i.e., government) bonds. Most other bonds have some kind of default risk—though even the U.S. Treasury is now rated to have some credit risk.

Q 6.9 With the revised probabilities:

1. The expected payoff is now $95\% \cdot \$210 + 1\% \cdot \$100 + 4\% \cdot \$0 = \$200.50$. Therefore, the expected rate of return is $\$200.50/\$200 = 0.25\%$.
2. You require an expected payoff of \$210 to expect to end up with 5%. Therefore, you must solve for a promised payment $95\% \cdot P + 1\% \cdot \$100 + 4\% \cdot \$0 = \$210 \Rightarrow P = \$209/0.95 = \$220$. On a loan of \$200, this is a 10% promised interest rate.

Q 6.10 No, the expected default premium is zero by definition.

Q 6.11 Both. The historical evidence is that lower-grade borrowers both default more often and pay less upon default.

Q 6.12 The actual cash flow is replaced by the expected cash flow, and the actual rate of return is replaced by the expected rate of return.

Q 6.13 The factory's expected value is $\mathsf{E}\left(\text{Value at Time 2}\right) = [0.5 \cdot \$500{,}000 + 0.5 \cdot \$1{,}000{,}000] = \$750{,}000$. Its present value is therefore $\$750{,}000/1.06^2 \approx \$667{,}497.33$.

Q 6.14 For the dynamite/dud project:

1. The expected payoff is $\mathsf{E}\left(P\right) = 20\% \cdot \$20{,}000 + 70\% \cdot \$40{,}000 + 10\% \cdot \$80{,}000 = \$40{,}000$.
2. The present value of the expected payoff is $\$40{,}000/1.08 \approx \$37{,}037$.
3. The three rate of return outcomes are $\$20{,}000/\$37{,}037 - 1 \approx -46\%$, $\$40{,}000/\$37{,}037 - 1 \approx +8\%$, $\$80{,}000/\$37{,}037 - 1 \approx +116\%$.
4. The expected rate of return is $20\% \cdot (-46\%) + 70\% \cdot (+8\%) + 10\% \cdot (+116\%) \approx 8\%$.

Q 6.15 No! Partners would share payoffs proportionally, not according to "debt comes first." For example, if it rains, the 6.7% partner would still receive \$4, and not \$0 that the levered equity owner would receive.

Q 6.16 To finance 80% of a \$75 building, the mortgage has to provide \$60 today. Start with the payoff table that contains what you know:

Event	Prob	Building	80% Mortgage	Levered
Rain	1/4	\$60	\$60	\$0
Sun	3/4	\$100	x	\$100-x
$E(V)$, Time 1		\$90	y	\$90-y
PV, Time 0		\$75	\$60	\$15
$E(r_{0,1})$		20%	20%	20%

In this interest environment, a mortgage that has a value of \$60 today must have an expected value of $y = \$60 \cdot (1 + 20\%) = \$72$. \$60 next year are worth \$50 today. Thus, $1/4 \cdot \$50 + 3/4 \cdot x = \$60$, which tells you that the promise to pay must be $x = \$76$.

Q 6.17 The text worked out the rates of return in the case of the junk mortgage. The previous question worked out the rates of return in the case of the solid mortgage.

	Rain	Sun	Expected
Junk Mortgage (\$70)	–14.3%	+31.4%	20%
Junk Equity (\$70)	–100.0%	+60.0%	20%
Solid Mortgage (\$60)	0%	+26.7%	20%
Solid Equity (\$60)	–100%	+60.0%	20%

The junk mortgage is indeed riskier than the solid mortgage. The junk equity is no riskier than the solid equity (though in a more general example, it would be). The building is the same building, and thus its risk has not changed.

Q 6.18

$$E(\text{ Payoff(} \$80 \le \text{Loan Promise} = x \le \$90) \text{)}$$
$$= \$60 + \$8 + \$6 + 40\% \cdot (x - \$80)$$

Q 6.19 The relevant line segment (and numeric answer) are $E = \$68 + 60\% \cdot (\$72 - \$70) = \$69.20$.

Q 6.20 The \$65 today requires an expected payoff of $1.2 \cdot \$65 = \$78$. This is on the final line segment. The formula is

$$E(\text{ Payoff(} \$90 \le \text{Loan Promise} = x \le \$100) \text{)}$$
$$= \$60 + \$8 + \$6 + \$4 + 20\% \cdot (x - \$90)$$
$$= \$78 + 20\% \cdot (x - \$90)$$

Thus, $x = \$90$.

Q 6.21 With infinitely many possible outcomes, the function of expected payoffs would be a smooth increasing function. For the mathematical nitpickers: [a] We really should not allow a normal distribution, because the value of the building cannot be negative; [b] The function would increase monotonically, but it would asymptote to an upper bound.

Q 6.22 With 20% probability, the loan will pay off \$20,000; with 80% probability, the loan will pay off the full promised \$40,000. Therefore, the loan's expected payoff is $20\% \cdot \$20{,}000 + 80\% \cdot \$40{,}000 = \$36{,}000$. The loan's price is $\$36{,}000/1.06 \approx \$33{,}962$. Therefore, the promised rate of return is $\$40{,}000/\$33{,}962 - 1 \approx 17.8\%$. The expected rate of return was given: 6%.

End of Chapter Problems

Q 6.23. Is this morning's CNN forecast of tomorrow's temperature a random variable? Is tomorrow's temperature a random variable?

Q 6.24. Does a higher reward (expected rate of return) always come with more risk?

Q 6.25. Would a single individual be effectively more, equally, or less risk-averse than a pool of such investors?

Q 6.26. A bond will pay off \$100 with a probability of 99% and will pay off nothing with a probability of 1%. The equivalent risk-free rate of return is 5%. What is an appropriate promised yield on this bond?

Q 6.27. An L.A. Lakers bond promises an investment rate of return of 9%. Time-equivalent Treasuries offer 6%. Is this necessarily a good investment? Explain.

Q 6.28. A Disney bond promises an investment rate of return of 7%. Time-equivalent Treasuries offer 7%. Is the Disney bond necessarily a bad investment? Explain.

Q 6.29. Using information from a current newspaper or the WWW, what is the annualized yield on corporate bonds (high-quality, medium-quality, high-yield) today?

Q 6.30. What are the main bond rating agencies and the meanings of their ranking categories? Roughly, what are the 10-year default rate differences between investment-grade and non-investment grade bonds this month?

Q 6.31. How is a credit swap like an insurance contract? Who is the insurer in a credit swap? Why would anyone want to buy such insurance?

Q 6.32. Debt is usually safer than equity. Does the risk of the rate of return on equity go up if the firm takes on more debt, *provided* the debt is low enough to remain risk-free? Illustrate with an example that you make up.

Q 6.33. A financial instrument will pay off as follows:

Prob	50%	25%	12.5%	6.25%	3.125%	3.125%
Payoff	$100	$110	$130	$170	$250	$500

Assume that the risk-free interest rate is 0.

1. What price today would make this a fair bet?
2. What is the maximum price that a risk-averse investor would be willing to pay?

Q 6.34. Now assume that the financial instrument from Q 6.33 costs $100.

1. What is its expected rate of return?
2. If the prevailing interest rate on time-equivalent Treasuries is 10%, and if financial default happens either completely (i.e., no repayment) or not at all (i.e., full promised payment), then what is the probability p that the security will pay off? In other words, assume that full repayment occurs with probability p and that zero repayment occurs with probability $1-p$. What is the p that makes the expected rate of return equal to 10%?

Q 6.35. Go to the Vanguard website. Look at funds by asset class, and answer this question for bond funds.

1. What is the current yield-to-maturity of a taxable Vanguard bond fund invested in Treasuries?
2. What is the current yield-to-maturity of a taxable Vanguard bond fund invested in investment-grade bonds?
3. What is the current yield-to-maturity of a taxable Vanguard bond fund invested in high-yield bonds?

Q 6.36. Return to the example on Page 113, but assume that the probability of receiving full payment of $210 in one year is only 95%, the probability of receiving $100 is 4%, and the probability of receiving absolutely no payment is 1%. If the bond quotes a rate of return of 12%, what is the time premium, the default premium, and the risk premium?

Q 6.37. A project costs $19,000 and promises the following cash flows:

	Y1	Y2	Y3
Cash Flows	$12,500	$6,000	$3,000

The appropriate discount rate is 15% per annum. Should you invest in this project?

Q 6.38. A bond promises to pay $12,000 and costs $10,000. The promised discount on equivalent bonds is 25% per annum. Is this bond a good deal?

Q 6.39. Assume that the probability that the Patriots will win the Superbowl is 55%. A souvenir shop outside the stadium will earn net profits of $1.5 million if the Patriots win and $1.0 million if they lose. You are the loan officer of the bank to whom the shop applied for a loan. You can assume that your bank is risk-neutral and that the bank can invest in safe projects that offer an expected rate of return of 10%.

1. What interest rate would you quote if the owner asked you for a loan for $900,000 today?
2. What interest rate would you quote if the owner asked you for a loan for $1,000,000 today?

(These two questions require that you compute the amount that you would demand for repayment.)

Q 6.40. A new project has the following probabilities:

	Failure	Success	Buyout
Prob	10%	85%	5%
Payoff (in millions)	$50	$200	$400

Assume risk neutrality. If a bond with $100 face value collateralized by this project promises an interest rate of 8%, then what is the prevailing cost of capital, and what do shareholders receive if the buyout materializes?

Q 6.41. Assume that the correct future cash flow is $100 and the correct discount rate is 10%. Consider the value effect of a 5% error in cash flows and the effect of a 5% error (50bp) in discount rates.

1. Graph the valuation impact (both in absolute values and in percent of the correct upfront present value) as a function of the number of years from one year to twenty years.
2. Is this an accurate real-world representation of how your uncertainty about your own calculations should look? In other words, is it reasonable to assume a 5% error for cash flows in twenty years? For the appropriate discount-rate applicable to twenty-year cash flows?

Q 6.42. Under risk neutrality, a factory can be worth $500,000 or $1,000,000 in *two* years, depending on product demand, each with equal probability. The appropriate cost of capital is 6% per year. The factory can be financed with proceeds of $500,000 from loans today. What are the promised and expected cash flows and rates of return for the factory (without a loan), the loan, and the levered factory owner?

Data and Programming for Masters Students

Statistics are often more intuitive to programmers than to non-programmers. Summation notation translates easily:

$$\sum_{i=1}^{n} x_i \quad \Leftrightarrow \quad \texttt{sum=0; for i=1 to n sum=sum+x[i]}$$

Even better, it is easy to program the drawing of random samples from statistical distributions. This makes it easy to test whether statistical methods (also called "estimators") really work as expected.

Task A: Write an R program that draws N random values from a normal distribution with mean 0.01 and standard deviation 0.10. (Incidentally, this is not far off from the monthly historical rates of return for many publicly traded stocks.) Bin your values and then plot them into a graph. How does this histogram look like?

Task B: Repeat Task A, but group your random draws into bunches of 12 and calculate the compounded rates of return. What are the mean and standard deviations of your draws (you are working with N/12 samples)? Does it matter whether you truncate normally distributed rates of returns at –100? If so, do it!

Repeat this with bunches of 60 and 120.

Task C: Under this process, if you could buy insurance on annual rates of return for \$0.10 that pays off \$1 if the rate of return is less than –20%, would you like this if you are risk-neutral?

Task D: Portfolio A has a monthly average rate of return of 5% with a standard deviation of 40%. Portfolio B has a monthly average rate of return of 4% with a standard deviation of 5%. What are your very long term expected buy-and-hold (i.e., geometric) rates of return for the two portfolios?

Task E: A portfolio has a mean average rate of return of 10% per period, and a standard deviation of x%. Assume that realized returns are truncated at –95%. That is, each draw below –95% is set to –95%. (Alternatively, draw from a log-normal distribution.) Now graph the difference between its arithmetic and geometric average rates of return as a function of x.

Task F: Assume you have a project whose expected rate of return is 20% with a standard deviation of 3% (and a truncated worst rate of return of –100%). Assume the world is risk-neutral and fair. Plot the *promised* rate of return on debt as a function of the project's leverage ratio.

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