Time Value of Money
and
Basic Capital Budgeting

(Welch, Chapter 02)

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Did you bring your calculator? Did you read these notes and the chapter ahead of time?

Can you add rates of return and/or interest rates?

If the bank posts an 8% interest rate and you invest \$100, how much money will you have at the end of the year?

Perfect Markets

For the next few chapters, we pretend we live in a **perfect market**. A perfect market must satisfy four assumptions:

1. No differences in opinion.
    - Uncertainty is ok, but everyone must agree to exactly what it is. We must not have different information or opinions.
2. No taxes.
    - or government interference or regulation [except perfect property rights].
3. No transaction costs.
    - Neither direct nor indirect.
4. No big sellers/buyers.
    - There must always be more where they came from. No (few) investors or firms are special. If investors differ, there must be infinitely many clones competing.

Why? What do perfect markets do for you?

- It is the simplest world. Analysis is easiest. Life is tough enough even in a perfect market.
- Any logic that fails in this simplest of worlds is surely wrong elsewhere, too. It will be wrong in a more realistic world, too.
  - Put differently, as your world gets closer and closer to perfection, your methods need to become closer and closer to what must hold in a perfect market.
- In Chapters 12 and 13, you will learn how our rules have to change (become more difficult, complex, and general) when the financial market is not perfect.
  - You will learn that the perfect market makes borrowing and lending rates equal and allows for a unique price for goods. Without it, we are often just toast.

Further Assumption, Only For Starting Exposition

- In **this** chapter, we further assume **perfect certainty**.
  - We *know* what the rates of return on every project are and will be. We don't need to worry about statistics and attitudes towards risk. All same-period (interest) rates of returns are the same.
- In **this** chapter, we also assume equal rates of returns per period.
  - No worries about yieldcurve (Chapter 5). A 1-year bond offers the same annualized [to be explained] rate of return as a 30-year bond.

Notation

- Time Convention:
  - $0 =$ Today, Right Now.
  - $1 =$ Next period (e.g., day, year, etc.)
  - $t =$ some time period (in the future).
  - $T =$ often to denote a final time period.
- **Flow**s pertain to something accumulating over a time span, e.g., a rate of return from $t-1$ to t. Opposite: instant moment quantity.
  - C or CF $=$ cash amount.
  - $C_t =$ instant cash amount at time t.
  - $D_{t-1,t} =$ a flow of D (e.g., dividends) over a time period $t-1$ to t.
  - D_t, common casual notation for $D_{t-1,t}$.
  - $D_{15,20}$, a flow of D (e.g., dividends) from time 15 through 20.
  - Return vs. Net Return vs. Rate of Return. (Interest rate.)
  - r, r_1, $r_{15,20}$, r_8: rates of return.

Small Notes

- This is a bit inconsistent. Dividends are really also paid at one instant in time, and thus should not be subscripted like a flow.
- If the investment is a loan, the rate of return is usually called an interest rate. We will (almost always) use the name "rate of return" and "interest rate" interchangeably.
- Although there is a difference between a return (CF_1), a net return ($\text{CF}_1 - \text{CF}_0$), and a (net) rate of return ($(\text{CF}_1 - \text{CF}_0)/\text{CF}_0$), in conversations, it is rarely explicit. Usually, you are assumed to know what the speaker means.

Rate of Return

The rate of return from investing CF_0 today and getting CF_1 at time 1 is

$$r = r_{0,1} = \frac{(CF_1 - CF_0)}{CF_0} = \frac{CF_1}{CF_0} - 1 \qquad .$$

This is the main formula of finance. With dividends D (or coupons or rent) paid at the *end* of the period (thus not reinvestable):

$$r = r_{0,1} = \frac{(CF_1 + D_{0,1} - CF_0)}{CF_0} = \frac{(CF_1 + D_{0,1})}{CF_0} - 1 \qquad .$$

Using our abbreviations,

$$r = r_1 = \frac{(CF_1 + D_1 - CF_0)}{CF_0} = \frac{(CF_1 + D_1)}{CF_0} - 1 \qquad .$$

- The dividend (or coupon or interim payment) yield is $D_{0,1}/CF_0$.
- The capital gain is $CF_1 - CF_0$.
- The percent price change is $(CF_1 - CF_0)/CF_0$.
- The (total) rate of return is the percent price change plus the interim payment yield.

If halfway through the course I casually write $r = P_1/P_0 - 1$ to describe the rate of return, then I am assuming that your P_1 includes any interim payments.

If the rate of return is positive, can the percent price change be negative?

If you invest \$5 and will receive \$8 in 10 years, what is your (holding) rate of return?

Can a rate of return be negative?

Can an interest rate be negative?

What is the prevailing interest rate today?

Compare 10% to 5%.

- Would you say that 10% is 5% more than 5%?
- Would you say that 10% is 100% more than 5%?

If you invest \$55,000 at an interest rate of 350 basis points above the 5% interest rate, what will you receive at the end of the period?

If you have \$5 and you earn a rate of return of 250%, how much will you have?

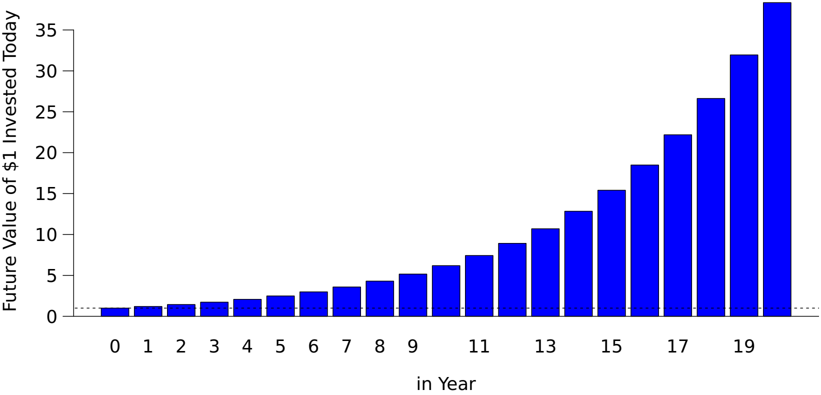
If you have \$5 and you earn a rate of return of 40%, how much money will you have?

What is the formula for the FV (Future Value) of money? How does it relate to the rate of return formula?

If you have \$5 and you earn a rate of return of 20% in the first year and a rate of return of 20% the following year, how much money will you have?

If you have \$5 and you earn a rate of return of 20% in each year, how much money will you have in x years?

Compounding at 20% Rate of Return Per Year



If the 1-year interest rate is 20% this year, how much money will you get for a \$500 investment today in one year? If the following 1-year interest will be 50%, how much money will you have after 2 years?

What is your total holding rate of return? Is it $50\% + 20\% = 70\%$?

What is the formula for the total holding rate of return, given the two individual rates of return?

What is the compounding formula to get $r_{0,x}$?

If your bank pays you 50% per year, what is your rate of return after 2 years?

You have \$100. You invest half each in two firms. Firm 1 makes 10% this month. Firm 2 makes 20% this month. How much did your portfolio make in total? Hint: $1.2 \cdot 1.1 = 1.31$.

Can you guess what people mean by the “cross-term” in the compounding formula?

If the 1-month interest rate is 1%, what is the 1 year rate?

If the 1-day rate is 0.02%, what is the 7-day (weekly) rate?

How good an approximation is simply adding interest? (covered again below.)

Algebra

$$x^a = b \iff x = b^{1/a}$$

$$a^x = b \iff x = \frac{\log b}{\log a}$$

A project for \$200 promises to return 8% per year. How much will you have after one month?

If the annual interest rate is 14%, what is the daily rate?

The monthly interest rate is 1.5%. There are 30.4 days in the average month. What is the weekly rate? Are there different ways to calculate it?

If you are doubling your money every 12 years, what is your rate of return per year?

If a project promises to return 8% per year, how long will it take for you to double your money?

Is compounding more like “adding” or “averaging”?

Rule of Thumb

If both the interest rate and the number of time periods is small,

$$(1 + r_n)^t \approx 1 + t \cdot r_n$$

Adding up instead of compounding gets to be a worse approximation if time increases and if the interest rate increases. (It also matters how much money is at stake.)

Warning: Conventions and Jargon

In principle, interest rates (and quotes) are not difficult—but they are tedious and often confusing, because everyone computes and quotes them slightly differently. Sometimes, it is obvious what people mean, sometimes interest rates are intentionally obscure in order to deceive you. You should know what you are talking about. Ask if you are unclear! There can be a lot of money at stake! Arbitrage desks on WS make most of their money on spreads below 20 basis points!

A bank quotes you 8% interest per year. If you invest \$1 million in the bank, what will you end up with?

Interest Quotes (Not Rates)

Unfortunately, many institutions give you interest "quotes," rather than interest rates, and the two are easy to confuse. This is especially bad with annualized interest quotes. There are many "pseudo interest rates" which are really "interest quotes" and not true "interest rates."

For example, sometimes banks and lenders calculate and pay daily interest rates, although they only credit the interest payment to accounts once per month. Such banks' daily interest rate calculation is the quoted annual interest rate divided by 365 ($r_d = r_y/365$). (Note: some banks use 360 days.)

Some banks quote for clarity

Interest rate: 8% compounded daily. Effective annual yield: 8.33%.

Is the 8% posted by the bank a true annual interest rate?

Is the "effective annual yield" a true annual interest rate?

Quotes vs. Rates: Government Bonds.

At US Treasury auctions, the government sells Treasury bills that pay \$10,000 in 180 days. If the government discount quote is 10% (which is absolutely *not* an interest rate), then it means you can purchase the Treasury bill at the auction for \$9,500, because they use the formula

$$\text{TB Price} = \$100 \cdot [100 - (\text{days to maturity}/360) \cdot \text{discount quote}]$$

$$= \$100 \cdot [100 - (180/360) \cdot \mathbf{10}] = \$9,500$$

Do not bother remembering this formula. This is hairy stuff—not conceptually, but detail-wise. If you are not going into to become a bond trader, you just need to know that it exists. I do not remember this formula, either. I looked it up. Heck, it could even have a mistake in it. The point is the point, not the detail **in this class**.

Financial newspapers and websites often print "95" instead of "9,500," because it is shorter, and T-bills are quoted in units of 100.

Assume the Treasury quote is indeed 95. If you invest \$1, how much will you receive in 6 months? 95%?

The Most Important Concept in (Corporate) Finance

Present Value

If you will receive \$7 next year, and the prevailing interest rate (or [opportunity] *cost of capital*) for investing in any type of project is 40%, what do you value this "\$7 next year" as of today?

What is your formula? How does your formula relate to the basic rate of return formula?

If you will receive \$7 in two years, and the prevailing (alternative) interest rate [or cost of capital] is 40%/year, what do you value this \$7 as of today?

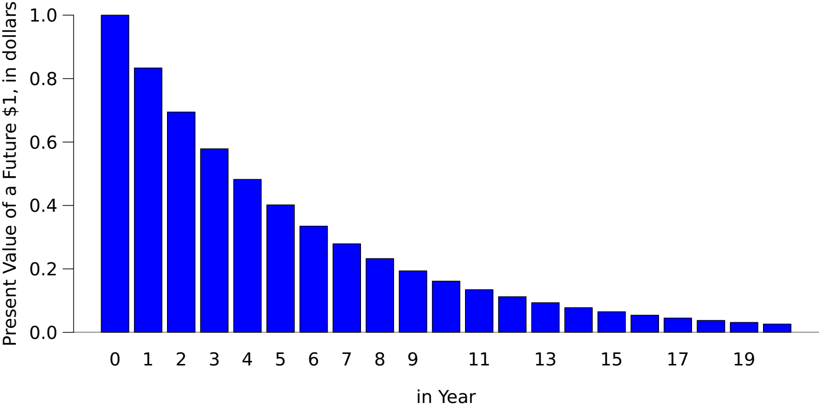
Given cash flow CF_t at time t, what is the PV formula?

The present value of cash CF_t at time t is

$$PV = CF_0 = \frac{CF_t}{(1 + r_{0,t})} \qquad .$$

- The quantity $1/(1 + r_{0,t})$ is called the **discount factor**. It is the factor that is multiplied to a future cash flow in order to obtain the future cash flow's current value.
- The quantity $r_{0,t}$ is called the discount rate, because it is the interest rate that is used to obtain the discount factor.
- In this context, the discount rate is also often called the **(opportunity) cost of capital**, because you should think of it either representing your alternative investment opportunities (if you have money) or your cost of borrowing (if you need money).
    - In our perfect market, the two are the same. That is, in our financial markets, you can invest into infinitely many alternatives for a rate of return that is exactly to your cost of borrowing.
- NPV simply means include the time-0 cash flow, often a cost (negative).

Discounting at 20% Rate of Return Per Year



How does the price of a bond change if the economy-wide interest rate changes?

If you will receive \$7 next year and another \$7 in two years, and the prevailing (alternative) interest rate [or cost of capital] is 40%, do you have the equivalent of \$14?

If you will receive $7 next year and another $7 in two years, and the prevailing (alternative) interest rate [or cost of capital] is 40% per year, what would you value this project as of today? What formula are you using?

If this project costs \$8, should you take this project?

If the cost of capital were 80%/year, should you take this project?

NPV Formula

The net present value is

$$\text{NPV} = \text{CF}_0 + \frac{\text{CF}_1}{(1+r_{0,1})} + \frac{\text{CF}_2}{(1+r_{0,2})} + \ldots$$

$$= \sum_{t=0}^{\infty} \frac{\text{CF}_t}{(1+r_{0,t})} \qquad .$$

It is called "net," because the first cash flow CF_0 is often negative.

In a perfect world, you should take all positive NPV projects. This is called the NPV capital budgeting rule.

The logical foundation of the NPV Rule

Here is how a perfect world w/o uncertainty must work:

- The NPV rule is optimal (other rules leave money on the table),
- and positive NPV projects must be scarce,

The proof is trivial. For example, presume that, in our perfect market, you can borrow or lend money at 8% anywhere today. The NPV formula says you will not make money on projects that cost \$1 today and yield \$1.08 next year. It says you should take all projects that yield more than \$1.08 next year. Now, presume that you have (infinitely) many investment opportunities that cost \$0.99 and yield \$1.08. (The NPV is positive.)

How would you get rich? Borrow \$0.99 and use it to buy the project. Tomorrow, you pay \$1.07, and receive \$1.08. You earn \$0.01. If you prefer money today, borrow against the \$0.01, or borrow \$1.00 to begin with.

If such projects are in limited supply, you (and everyone else) would buy up all such projects, until the project's equilibrium price has increased to make the project zero NPV.

(If you can short projects, and you have willing buyers for negative NPV projects, you can just sell them and thereby invert the argument.)

Is a good stock or good firm a good investment?
Is a bad stock or bad firm a bad investment?

Are fast-growing firms better investments than slow-growing firms?