2

Present Value

The Mother of All Finance

We begin with the concept of a rate of return—the cornerstone of finance. You can always earn an interest rate (and interest rates are rates of return) by depositing your money today into the bank. This means that money today is more valuable than the same amount of money next year. This concept is called the *time value of money* (TVM)—$1 in present value is better than $1 in future value.

Investors make up just one side of the financial markets. They give money today in order to receive money in the future. Firms often make up the other side. They decide what to do with the money—which projects to take and which projects to pass up—a process called *capital budgeting*. You will learn that there is one best method for making this critical decision. The firm should translate all *future* cash flows—both inflows and outflows—into their equivalent *present values* today. Adding in the cash flow today gives the *net present value*, or NPV. The firm should take all projects that have positive net present values and reject all projects that have negative net present values.

This all sounds more complex than it is, so we'd better get started.

2.1 The Basic Scenario

As promised, we begin with the simplest possible scenario. In finance, this means that we assume that we are living in a so-called **perfect market**:

We start with a so-called perfect market.

- There are no taxes.
- There are no transaction costs (costs incurred when buying and selling).
- There are no differences in information or opinions among investors (although there can be risk).
- There are so many buyers and sellers (investors and firms) in the market that the presence or absence of just one (or a few) individuals does not have an influence on the price.

The perfect market allows us to focus on the basic concepts in their purest forms, without messy real-world factors complicating the exposition. We will use these assumptions as our sketch of how financial markets operate, though not necessarily how firms' product markets work. You will learn in Chapter 11 how to operate in a world that is not perfect. (This will be a lot messier.)

In this chapter, we will make three additional assumptions (that are not required for a market to be considered "perfect") to further simplify the world:

In early chapters only, we add even stronger assumptions.

- The interest rate per period is the same.

- There is no inflation.
- There is no risk or uncertainty. You have perfect foresight.

Of course, this financial utopia is unrealistic. However, the tools that you will learn in this chapter will also work in later chapters, where the world becomes not only progressively more realistic but also more difficult. Conversely, if any tool does not give the right answer in our simple world, it would surely make no sense in a more realistic one. And, as you will see, the tools have validity even in the messy real world.

Q 2.1. What are the four perfect market assumptions?

2.2 Loans and Bonds

Finance jargon: interest, loan, bond, fixed income, maturity.

The material in this chapter is easiest to explain in the context of bonds and loans. A **loan** is the commitment of a borrower to pay a predetermined amount of cash at one or more predetermined times in the future (the final one called **maturity**), usually in exchange for cash upfront today. Loosely speaking, the difference between the money lent and the money paid back is the **interest** that the lender earns. A **bond** is a particular kind of loan, so named because it "binds" the borrower to pay money. Thus, for an investor, "buying a bond" is the same as "extending a loan." Bond buying is the process of giving cash today and receiving a binding promise for money in the future. Similarly, from the firm's point of view, it is "giving a bond," "issuing a bond," or "selling a bond." Loans and bonds are also sometimes called **fixed income** securities, because they promise a fixed amount of payments to the holder of the bond.

Why learn bonds first? Because they are easiest.

You should view a bond as just another type of investment project—money goes in, and money comes out. You could slap the name "corporate project" instead of "bond" on the cash flows in the examples in this chapter, and nothing would change. In Chapter 5, you will learn more about Treasuries, which are bonds issued by the U.S. Treasury. The beauty of such bonds is that you know exactly what your cash flows will be. (Despite Washington's dysfunction, we will assume that our Treasury cannot default.) Besides, much more capital in the economy is tied up in bonds and loans than is tied up in stocks, so understanding bonds well is very useful in itself.

Interest rates: limited upside. Rates of return: arbitrary upside.

You already know that the net return on a loan is called interest, and that the rate of return on a loan is called the **interest rate**—though we will soon firm up your knowledge about interest rates. One difference between an interest payment and a noninterest payment is that the former usually has a maximum payment, whereas the latter can have unlimited upside potential. However, not every rate of return is an interest rate. For example, an investment in a lottery ticket is not a loan, so it does not offer an interest rate, just a rate of return. In real life, its payoff is uncertain—it could be anything from zero to an unlimited amount. The same applies to stocks and many corporate projects. Many of our examples use the phrase "interest rate," even though the examples almost always work for any other rates of return, too.

Bond: defined by payment next year. Savings: defined by deposit this year.

Is there any difference between buying a bond for $1,000 and putting $1,000 into a bank savings account? Yes, a small one. The bond is defined by its future promised payoffs—say, $1,100 next year—and the bond's value and price today are based on these future payoffs. But as the bond owner, you know exactly how much you will receive next year. An investment in a bank savings account is defined by its investment today. The interest rate can and will change every day, so you do not know what you will end up with next year. The exact amount depends on future interest rates. For example, it could be $1,080 (if interest rates decrease) or $1,120 (if interest rates increase).

If you want, you can think of a savings account as a sequence of consecutive 1-day bonds: When you deposit money, you buy a 1-day bond, for which you know the interest rate this one day in advance, and the money automatically gets reinvested tomorrow into another bond with whatever the interest rate will be tomorrow.

A bank savings account is like a sequence of 1-day bonds.

Q 2.2. Is a deposit into a savings account more like a long-term bond investment or a series of short-term bond investments?

A Question of Principal

Who were the world's first financiers? Candidates are the Babylonian Egibi family (7th Century BCE), the Athenian Pasion (4th), or many Ancient Egyptians (1st). The latter even had a check-writing system! Of course, moneylenders were never popular—a fact that readers of the New Testament or the Koran already know. In medieval Europe, Genoa was an early innovator. In 1150, it issued a 400-lire 29-year bond, collateralized by taxes on market stalls. By the 15th Century, the first true modern banks appeared, an invention that spread like wildfire throughout Europe. *The Economist, Jan 10, 2009*

2.3 Returns, Net Returns, and Rates of Return

Defining return and our time. Our convention is that 0 means "right now."

The most fundamental financial concept is that of a return. The payoff or (dollar) **return** of an investment is simply the amount of cash (C) it returns. For example, an investment project that returns $12 at time 1 has

$$C_1 = \text{Cash Return at Time 1} = \$12$$

This subscript is an instant in time, usually abbreviated by the letter t. When exactly time 1 occurs is not important: It could be tomorrow, next month, or next year. But if we mean "right now," we use the subscript 0.

Defining net return and rate of return.

The net payoff, or **net return**, is the difference between the return and the initial investment. It is positive if the project is profitable and negative if it is unprofitable. For example, if the investment costs $10 today and returns $12 at time 1 with nothing in between, then it earns a net return of $2. Notation-wise, we need to use two subscripts on returns—the time when the investment starts (0) and when it ends (1).

$$\text{Net Return from Time 0 to Time 1} = \$12 - \$10 = \$2$$

$$\text{Net Return}_{0,1} = C_1 - C_0$$

The double subscripts are painful. Let's agree that if we omit the first subscript on flows, it means zero. The **rate of return**, usually abbreviated r, is the net return expressed as a percentage of the initial investment.

$$\text{Rate of Return from Time 0 to Time 1} = \frac{\$2}{\$10} = 20\%$$

$$r_{0,1} = r_1 = \frac{\text{Net Return from Time 0 to Time 1}}{\text{Purchase Price at Time 0}}$$

Here, I used our new convention and abbreviated $r_{0,1}$ as r_1. Often, it is convenient to calculate the rate of return as

$$r_1 = \frac{\$12 - \$10}{\$10} = \frac{\$12}{\$10} - 1 = 20\%$$

$$r_1 = \frac{C_1 - C_0}{C_0} = \frac{C_1}{C_0} - 1 \quad (2.1)$$

Percent (the symbol %) is a unit of 1/100. So 20% is the same as 0.20.

Interest Rates over the Millennia

Historical interest rates are fascinating, perhaps because they look so similar to today's interest rates. Nowadays, typical interest rates range from 2% to 20% (depending on the loan). For over 2,500 years—from about the 30th century B.C.E. to the 6th century B.C.E.—normal interest rates in Sumer and Babylonia hovered around 10–25% per annum, though 20% was the legal maximum. In ancient Greece, interest rates in the 6th century B.C.E. were about 16–18%, dropping steadily to about 8% by the turn of the millennium. Interest rates in ancient Egypt tended to be about 10–12%. In ancient Rome, interest rates started at about 8% in the 5th century B.C.E. but began to increase to about 12% by the third century A.C.E. (a time of great upheaval and inflation). When lending resumed in the late Middle Ages (12th century), personal loans in continental Europe hovered around 10-20% (50% in England). By the Renaissance (16th Century), commercial loan rates had fallen to 5–15% in Italy, the Netherlands, and France. By the 17th century, even English interest rates had dropped to 6–10% in the first half, and to 3–6% in the second half (and mortgage rates were even lower). Most of the American Revolution was financed with French and Dutch loans at interest rates of 4–5%. *Homer and Sylla, A History of Interest Rates*

How to compute returns with interim payments. Capital gains versus returns.

Many investments have interim payments. For example, many stocks pay interim cash **dividends**, many bonds pay interim cash **coupons**, and many real estate investments pay interim **rent**. How would you calculate the rate of return then? One simple method is to just add interim payments to the numerator. Say an investment costs \$92, pays a dividend of \$5 (at the end of the period), and then is worth \$110. Its rate of return is

$$r = \frac{\$110 + \$5 - \$92}{\$92} = \frac{\$110 - \$92}{\$92} + \frac{\$5}{\$92} = 25\%$$

$$r_1 = \frac{C_1 + \text{All Dividends from 0 to 1} - C_0}{C_0} = \underbrace{\frac{C_1 - C_0}{C_0}}_{\text{Capital Gain, in \%}} + \underbrace{\frac{\text{All Dividends}}{C_0}}_{\text{Dividend Yield}}$$

When there are intermittent and final payments, then returns are often broken down into two additive parts. The first part, the price change or **capital gain**, is the difference between the purchase price and the final price, *not* counting interim payments. Here, the capital gain is the difference between \$110 and \$92, that is, the \$18 change in the price of the investment. It is often quoted in percent of the price, which would be \$18/\$92 or 19.6% here. The second part is the amount received in interim payments. It is the dividend or coupon or rent, here \$5. When it is divided by the price, it has names like **dividend yield**, **current yield**, **rental yield**, or **coupon yield**, and these are also usually stated in percentage terms. In our example, the dividend yield is $\$5/\$92 \approx 5.4\%$. Of course, if the interim yield is high, you might be experiencing a negative capital gain and still have a positive rate of return. For example, a bond that costs \$500, pays a coupon of \$50, and then sells for \$490, has a **capital loss** of \$10 (which comes to a –2% capital yield) but a rate of return of $(\$490 + \$50 - \$500)/\$500 = +8\%$. You will almost always work with rates of return, not with capital gains. The only exception is when you have to work with taxes, because the IRS treats capital gains differently from interim payments. (We will cover taxes in Section 11.4.)

➤ Corporate payouts and dividend yields, Chapter 20, Pg.555.

➤ Taxes on capital gains, Sect. 11.4, Pg.257.

Most of the time, people (incorrectly but harmlessly) abbreviate a rate of return or net return by calling it just a return. For example, if you say that the return on your $10,000 stock purchase was 10%, you obviously do not mean you received a unitless 0.1. You really mean that your rate of return was 10% and you received $1,000. This is usually benign, because your listener will know what you mean. Potentially more harmful is the use of the phrase *yield*, which, strictly speaking, means *rate of return*. However, it is often misused as a shortcut for dividend yield or coupon yield (the percent payout that a stock or a bond provides). If you say that the yield on your stock was 5%, then some listeners may interpret it to mean that you earned a total rate of return of 5%, whereas others may interpret it to mean that your stock paid a dividend yield of 5%.

People often use incorrect terms, but the meaning is usually clear, so this is harmless.

➤ Nominal, Sect. 5.2, Pg.82.

Interest rates should logically always be positive. After all, you can always earn 0% if you keep your money under your mattress—you thereby end up with as much money next period as you have this period. Why give your money to someone today who will give you less than 0% (less money in the future)? Consequently, interest rates are indeed almost always positive—the rare exceptions being both bizarre and usually trivial.

[Nominal] interest is [usually] nonnegative.

Here is another language problem: What does the statement "the interest rate has just increased by 5%" mean? It could mean either that the previous interest rate, say, 10%, has just increased from 10% to $10\% \cdot (1+5\%) = 10.5\%$, or that it has increased from 10% to 15%. Because this is unclear, the **basis point** unit was invented. A basis point is simply 1/100 of a percent. If you state that your interest rate has increased by 50 basis points, you definitely mean that the interest rate has increased from 10% to 10.5%. If you state that your interest rate has increased by 500 basis points, you definitely mean that the interest rate has increased from 10% to 15%.

Basis points avoid an ambiguity in the English language: 100 basis points equals 1%.

IMPORTANT

100 basis points constitute 1%. Somewhat less common, 1 point is 1%.
Points and basis points help with "percentage ambiguities."

Q 2.3. An investment costs $1,000 and pays a return of $1,050. What is its rate of return?

Q 2.4. An investment costs $1,000 and pays a net return of $25. What is its rate of return?

Q 2.5. Is 10 the same as 1,000%?

Q 2.6. You buy a stock for $40 per share today. It will pay a dividend of $1 next month. If you can sell it for $45 right after the dividend is paid, what would be its dividend yield, what would be its capital gain (also quoted as a capital gain yield), and what would be its total rate of return?

Q 2.7. By how many basis points does the interest rate change if it increases from 9% to 12%?

Q 2.8. If an interest rate of 10% decreases by 20 basis points, what is the new interest rate?

2.4 Time Value, Future Value, and Compounding

Time Value of Money = Earn Interest.

Because you can earn interest, a given amount of money today is worth more than the same amount of money in the future. After all, you could always deposit your money today into the bank and thereby receive more money in the future. This is an example of the **time value of money**, which says that a dollar today is worth more than a dollar tomorrow. This ranks as one of the most basic and important concepts in finance.

The Future Value of Money

Here is how to calculate future payoffs given a rate of return and an initial investment.

➤ Rate of Return, Formula 2.1, Pg.14.

How much money will you receive in the future if the rate of return is 20% and you invest \$100 today? Turn around the rate of return formula (Formula 2.1) to determine how money will grow over time given a rate of return:

$$20\% = \frac{\$120 - \$100}{\$100} \Leftrightarrow \$100 \cdot (1 + 20\%) = \$100 \cdot 1.2 = \$120$$
$$r_1 = \frac{C_1 - C_0}{C_0} \Leftrightarrow C_0 \cdot (1 + r_1) = C_1$$

The \$120 next year is called the **future value** (**FV**) of \$100 today. Thus, future value is the value of a present cash amount at some point in the future. It is the time value of money that causes the future value, \$120, to be higher than its present value (PV), \$100. Using the abbreviations FV and PV, you could also have written the above formula as

$$r_1 = \frac{FV - PV}{PV} \Leftrightarrow FV = PV \cdot (1 + r_1)$$

➤ Apples and Oranges, Sect. 5.2, Pg.82.

(If we omit the subscript on the r, it means a 1-period interest rate from now to time 1, i.e., r_1.) Please note that the time value of money is not the fact that the prices of goods may change between today and tomorrow (that would be inflation). Instead, the time value of money is based exclusively on the fact that your money can earn interest. Any amount of cash today is worth more than the same amount of cash tomorrow. Tomorrow, it will be the same amount plus interest.

Q 2.9. A project has a rate of return of 30%. What is the payoff if the initial investment is \$250?

Compounding and Future Value

Interest on interest (or rate of return on rate of return) means rates cannot be added.

Now, what if you can earn the same 20% year after year and reinvest all your money? What would your two-year rate of return be? Definitely *not* 20% + 20% = 40%! You know that you will have \$120 in year 1, which you can reinvest at a 20% rate of return from year 1 to year 2. Thus, you will end up with

$$C_2 = \$100 \cdot (1 + 20\%)^2 = \$100 \cdot 1.2^2 = \$120 \cdot (1 + 20\%) = \$120 \cdot 1.2 = \$144$$
$$C_0 \cdot (1 + r)^2 = C_1 \cdot (1 + r) = C_2$$

This \$144—which is, of course, again a future value of \$100 today—represents a total two-year rate of return of

$$r_2 = \frac{\$144 - \$100}{\$100} = \frac{\$144}{\$100} - 1 = 44\%$$
$$\frac{C_2 - C_0}{C_0} = \frac{C_2}{C_0} - 1 = r_2$$

This is more than 40% because the original net return of $20 in the first year earned an additional $4 in interest in the second year. You earn interest on interest! This is also called **compound interest**. Similarly, what would be your 3-year rate of return? You would invest $144 at 20%, which would provide you with

$$C_3 = \$144 \cdot (1+20\%) = \$144 \cdot 1.2 = \$100 \cdot (1+20\%)^3 = \$100 \cdot 1.2^3 = \$172.80$$
$$C_2 \cdot (1+r) = C_0 \cdot (1+r)^3 = C_3$$

The "one-plus" formula.

Your 3-year rate of return from time 0 to time 3 (call it r_3) would thus be

$$r_3 = \frac{\$172.80 - \$100}{\$100} = \frac{\$172.80}{\$100} - 1 = 72.8\%$$
$$\frac{C_3 - C_0}{C_0} = \frac{C_3}{C_0} - 1 = r_3$$

This formula translates the three sequential 1-year rates of return into one 3-year **holding rate of return**—that is, what you earn if you hold the investment for the entire period. This process is called **compounding**, and the formula that does it is the "one-plus formula":

$$(1 + 72.8\%) = (1+20\%) \cdot (1+20\%) \cdot (1+20\%)$$
$$(1 + r_3) = (1+r) \cdot (1+r) \cdot (1+r)$$

or, if you prefer it shorter, $1.728 = 1.2^3$.

Exhibit 2.1 shows how your $100 would grow if you continued investing it at a rate of return of 20% per annum. The function is exponential—that is, it grows faster and faster as interest earns more interest.

IMPORTANT

The compounding formula translates sequential future rates of return into an overall holding rate of return:

$$\underbrace{(1+r_t)}_{\text{Multiperiod Holding Rate of Return}} = \underbrace{(1+r)^t}_{\text{Multiperiod Holding Rate of Return}} = \underbrace{(1+r)}_{\text{Current 1-Period Spot Rate of Return}} \cdot \underbrace{(1+r)}_{\text{Next 1-Period Rate of Return}} \cdots \underbrace{(1+r)}_{\text{Final 1-Period Rate of Return}}$$

The first rate is called the spot rate because it starts now (on the spot).

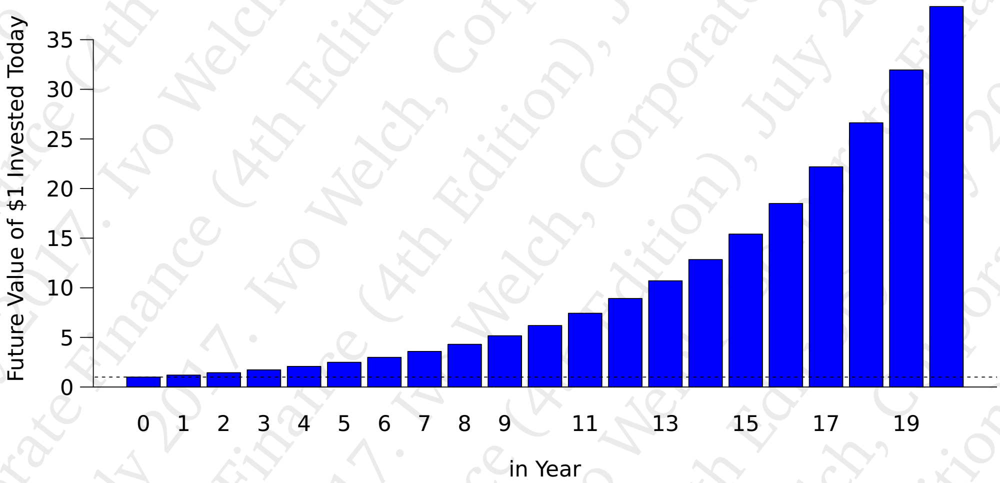
The compounding formula is so common that you must memorize it.

Another example of a payoff computation.

You can use the compounding formula to compute all sorts of future payoffs. For example, an investment project that costs $212 today and earns 10% each year for 12 years will yield an overall holding rate of return of

$$r_{12} = (1+10\%)^{12} - 1 = (1.1^{12} - 1) \approx 213.8\%$$
$$(1+r)^t - 1 = r_{12}$$

Your $212 investment today would therefore turn into a future value of



Period	Start value	1 + one-year rate	End value	Total factor from time 0	Total rate of return $r_{0,t} = (1+r)^t - 1$
0 to 1	$100	(1 + 20%)	$120.00	1.2	20.0%
1 to 2	$120	(1 + 20%)	$144.00	$1.2 \cdot 1.2 = 1.44$	44.0%
2 to 3	$144	(1 + 20%)	$172.80	$1.2 \cdot 1.2 \cdot 1.2 = 1.728$	72.8%
⋮					

Exhibit 2.1: *Compounding over 20 Years at 20% per Annum.* Money grows at a constant rate of 20% per annum. If you compute the graphed value at 20 years out, you will find that each dollar invested right now is worth $38.34 in 20 years. The money at first grows in a roughly linear pattern, but as more and more interest accumulates and itself earns more interest, the graph accelerates steeply upward.

$$C_{12} = \$212 \cdot (1 + 10\%)^{12} = \$212 \cdot 1.1^{12} \approx \$212 \cdot (1 + 213.8\%) \approx \$665.35$$
$$C_0 \cdot (1+r)^{12} \qquad = \qquad C_{12}$$

"Uncompounding": Turn around the formula to compute individual holding rates.

Now suppose you wanted to know what constant two 1-year interest rates (r) would give you a two-year rate of return of 50%. The answer is not 25%, because $(1 + 25\%) \cdot (1 + 25\%) - 1 = 1.25^2 - 1 = 56.25\%$. Instead, you need to solve

$$(1+r) \cdot (1+r) = (1+r)^2 = 1 + 50\% = 1.50$$

The correct answer is

$$r = \sqrt[2]{1 + 50\%} - 1 \approx 22.47\%$$
$$= \sqrt[t]{1 + r_t} - 1 = r$$

➤ Exponentiation, Book Appendix, Chapter A, Pg.621.

Check your answer: $(1 + 22.47\%) \cdot (1 + 22.47\%) = 1.2247^2 \approx (1 + 50\%)$. If the 12-month interest rate is 213.8%, what is the 1-month interest rate?

$$(1+r)^{12} \approx 1+213.8\%$$
$$\Leftrightarrow r = \sqrt[12]{1+213.8\%}-1 = (1+213.8\%)^{1/12}-1 \approx 10\%$$

Interestingly, compounding works even over fractional time periods. Say the overall interest rate is 5% per year, and you want to find out what the rate of return over half a year would be. Because $(1+r_{0.5})^2 = (1+r_1)$, you would compute

You can determine fractional time interest rates via compounding, too.

$$(1+r_{0.5}) = (1+r_1)^{0.5} = (1+5\%)^{0.5} \approx 1+2.4695\% = 1.024695$$

Check—compounding 2.4695% over two (6-month) periods indeed yields 5%:

$$(1+2.4695\%)\cdot(1+2.4695\%) = 1.024695^2 \approx (1+5\%)$$
$$(1+r_{0.5})\cdot(1+r_{0.5}) = (1+r_{0.5})^2 = (1+r_1)$$

Life Expectancy and Credit

Your life expectancy may be 80 years, but 30-year bonds existed even in an era when life expectancy was only 25 years—at the time of Hammurabi, around 1700 B.C.E. (Hammurabi established the Kingdom of Babylon and is famous for the Hammurabi Code, the first known legal system.) Moreover, four thousand years ago, Mesopotamians already solved interesting financial problems. A cuneiform clay tablet contains the oldest known interest rate problem for prospective students of the financial arts. The student must figure out how long it takes for 1 mina of silver, growing at 20% interest per year, to reach 64 minae. Because the interest compounds in an odd way (20% of the principal is accumulated until the interest is equal to the principal, and then it is added back to the principal), the answer to this problem is 30 years, rather than 22.81 years. This is not an easy problem to solve—and it even requires knowledge of logarithms!

William Goetzmann, Yale University

If you know how to use logarithms, you can also use the same formula to determine how long it will take at the current interest rate to double or triple your money. For example, at an interest rate of 3% per year, how long would it take you to double your money?

You need logs to determine the time needed to get x times your money.

$$(1+3\%)^x = (1+100\%) \Leftrightarrow x = \frac{\log(1+100\%)}{\log(1+3\%)} = \frac{\log(2.00)}{\log(1.03)} \approx 23.5$$
$$(1+r)^t = (1+r_t) \Leftrightarrow t = \frac{\log(1+r_t)}{\log(1+r)}$$

Compound rates of return can be negative even when the average rate of return is positive: think +200% followed by −100%. The average arithmetic rate of return in this example is (200% + (−100%))/2 = +50%, while the compound rate of return is −100%. Not a good investment! Thinking in arithmetic terms for wealth accumulation is a common mistake, if only because funds usually advertise their average rate of return. High-volatility funds (i.e., funds that increase and decrease a lot in value) look particularly good on this incorrect performance measure.

One more thing...

Errors: Adding or Compounding Interest Rates?

Adding rather than compounding can make forgivably small mistakes in certain situations—but don't be ignorant of what you are doing.

Unfortunately, when it comes to interest rates in the real world, many users are casual, sometimes to the point where they are outright wrong. Some people mistakenly add interest rates instead of compounding them. When the investments, interest rates, and time length are small, the difference between the correct and incorrect computation is often minor, so this practice can be acceptable, even if it is wrong. For example, when interest rates are 10%, compounding yields

$$(1+10\%) \cdot (1+10\%) - 1 = 1.1^2 - 1 = 21\%$$
$$(1+r) \cdot (1+r) - 1 = r_2$$
$$= 1 + r + r + r \cdot r - 1$$

which is not exactly the same as the simple sum of two r's, which comes to 20%. The difference between 21% and 20% is the "cross-term" $r \cdot r$. This cross-product is especially unimportant if both rates of return are small. If the two interest rates were both 1%, the cross-term would be 0.0001. This is indeed small enough to be ignored in most situations and is therefore a forgivable approximation. However, when you compound over many periods, you accumulate more and more cross-terms, and eventually the quality of your approximation deteriorates. For example, over 100 years, $1 million invested at 1% per annum compounds to $2.71 million, not to $2 million.

Q 2.10. If the 1-year rate of return is 20% and interest rates are constant, what is the 5-year holding rate of return?

Q 2.11. If you invest $2,000 today and it earns 25% per year, how much will you have in 15 years?

Q 2.12. What is the holding rate of return for a 20-year investment that earns 5%/year each year? What would a $200 investment grow into?

Q 2.13. A project lost one-third of its value each year for 5 years. What was its total holding rate of return? How much is left if the original investment was $20,000?

Q 2.14. If the 5-year holding rate of return is 100% and interest rates are constant, what is the (compounding) annual interest rate?

Q 2.15. What is the quarterly interest rate if the annual interest rate is 50%?

Q 2.16. If the per-year interest rate is 5%, what is the two-year total interest rate?

Q 2.17. If the per-year interest rate is 5%, what is the 10-year total interest rate?

Q 2.18. If the per-year interest rate is 5%, what is the 100-year total interest rate? How does this compare to 100 times 5%?

Q 2.19. At a constant rate of return of 6% per annum, how many years does it take you to triple your money?

IMPORTANT

When you compare your calculations to mine (not only in my exposition in the chapter itself but also in my answers to these questions in the back of the chapter), you will often find that they are slightly different. This is usually a matter of rounding precision—depending on whether you carry intermediate calculations at full precision or not. Such discrepancies are an unavoidable nuisance, but they are *not* a problem. You should check whether your answers are close, not whether they are exact to the x-th digit after the decimal point.

How Banks Quote Interest Rates

Banks add to the confusion, quoting interest rates using strange but traditional conventions.

Banks and many other financial institutions use a number of conventions for quoting interest rates that may surprise you. Consider the example of a loan or a deposit that has one flow of \$1,000,000 and a return flow of \$1,100,000 in six months. Obviously, the simple holding rate of return is 10%. Here is what you might see:

The effective annual rate (EAR) is what our book has called the real interest rate or holding rate of return. In this case, our only problem is to re-quote the six-month 10% rate into a twelve-month rate. This is easy,

$$\text{EAR} = (1 + 10\%)^{12/6} - 1 = 21\%$$

This 21% is usually a supplementary rate that any bank would quote you on both deposits and loans. The EAR is also sometimes called the **annual percentage yield (APY)**. And it is also sometimes (and ambiguously) called the **annual equivalent rate (AER)**.

The annual interest rate (stated without further explanations) is not really a rate of return, but just a method of quoting an interest rate. The true daily interest rate is this annual interest quote divided by 365 (or 360 by another convention). In the example, the 10% half-year interest rate translates into

$$\text{AIR} = \underbrace{(1.10^{1/(365/2)} - 1)}_{\text{Daily Interest Rate} \approx 0.0522384\%} \cdot 365 = 19.07\%$$

The annual interest rate is usually how banks quote interest rates on savings or checking accounts. Conversely, if the bank advertises a savings interest rate of 20%, any deposit would really earn an effective annual rate of $(1 + 20\%/365)^{365} - 1 \approx 22.13\%$ per year.

The annual percentage rate (APR) is a complete mess. Different books define it differently. Most everyone agrees that APR is based on monthly compounding:

$$\text{APR} = \underbrace{(1.10^{1/(12/2)} - 1)}_{\text{Monthly Interest Rate} \approx 1.6\%} \cdot 12 = 19.21\%$$

However, the APR is also supposed to include fees and other expenses. Say the bank charged \$10,000 in application and other fees. This is paid upfront, so we should recognize that the interest rate is not 10%, but $\$1,100,000/\$990,000 - 1 \approx 11.1\%$. We could then "monthly-ize" this holding rate of return into an APR of $(1.1111^{2/12} - 1) \cdot 12 \approx 21.26\%$.

So far, so good—except different countries require different fees to be included. In the United States, there are laws that state how APR should be calculated—and not just one, but a few (the Truth in Lending Act of 1968 [Reg Z], the Truth in Savings Act of 1991, the Consumer Credit Act of 1980, and who knows what other Acts). Even with all these laws, the APR is still not fully precise and comparable. To add insult to injury, the APR is also sometimes abbreviated as AER, just like the EAR.

Interest rates are not intrinsically difficult, but they can be tedious, and definitional confusions abound. So if real money is on the line, you should ask for the full and exact calculations of all payments in and all payments out, and not just rely on what *you* think it is that the bank is really quoting you. Besides, the above rates are not too interesting (yet), because they don't work for loans that have multiple payments. You have to wait for that until we cover the yield-to-maturity.

➤ Yield-To-Maturity, Sect. 4.2, Pg.59.

Let's look at a **certificate of deposit (CD)**, which is a longer-term investment vehicle than a savings account deposit. If your bank wants you to deposit your money in a CD, do you think it will put the more traditional interest rate quote or the APY on its sign in the window? Because the APY of 10.52% looks larger and thus more appealing to depositors than the traditional 10%

interest rate quote, most banks advertise the APY for deposits. If you want to borrow money from your bank, do you think your loan agreement will similarly emphasize the APY? No. Most of the time, banks leave this number to the fine print and focus on the APR (or the traditional interest rate quote) instead.

Q 2.20. If you earn an (effective) interest rate of 12% per annum, how many basis points do you earn in interest on a typical calendar day? (Assume a year has 365.25 days.)

Q 2.21. If the bank quotes an interest rate of 12% per annum (not as an effective interest rate), how many basis points do you earn in interest on a typical day?

Q 2.22. If the bank states an *effective* interest rate of 12% per annum, and there are 52.2 weeks per year, how much interest do you earn on a deposit of $1,000 over 1 week? On a deposit of $100,000?

Q 2.23. If the bank quotes interest of 12% per annum, and there are 52.2 weeks, how much interest do you earn on a deposit of $1,000 over 1 week?

Q 2.24. If the bank quotes interest of 12% per annum, and there are 52.2 weeks, how much interest do you earn on a deposit of $1,000 over 1 year?

Q 2.25. If the bank quotes an interest rate of 6% per annum, what does a deposit of $100 in the bank come to after one year?

Q 2.26. If the bank quotes a loan APR rate of 8% per annum, compounded monthly, and without fees, what do you have to pay back in one year if you borrow $100 from the bank?

2.5 Present Value, Discounting, and Capital Budgeting

Capital budgeting: Should you budget capital for a project?

Now turn to the flip side of the future value problem: If you know how much money you will have next year, what does this correspond to in value *today*? This is especially important in a corporate context, where the question is, "Given that Project X will return $1 million in 5 years, how much should you be willing to pay to undertake this project today?" The process entailed in answering this question is called **capital budgeting** and is at the heart of corporate decision making. (The origin of the term was the idea that firms have a "capital budget," and that they must allocate capital to their projects within that budget.)

The "present value formula" is nothing but the rate of return definition—inverted to translate future cash flows into (equivalent) today's dollars.

Start again with the rate of return formula

$$r_1 = \frac{C_1 - C_0}{C_0} = \frac{C_1}{C_0} - 1$$

You only need to turn this formula around to answer the following question: If you know the prevailing interest rate in the economy (r_1) and the project's future cash flows (C_1), what is the project's value to you *today*? In other words, you are looking for the **present value** (**PV**)—the amount a future sum of money is worth today, given a specific rate of return. For example, if the interest rate is 10%, how much would you have to save (invest) to receive $100 next year? Or, equivalently, if your project will return $100 next year, what is the project worth to you today? The answer lies in the present value formula, which translates future money into today's money. You merely need to rearrange the rate of return formula to solve for the present value:

$$C_0 = \frac{\$100}{1 + 10\%} = \frac{\$100}{1.1} \approx \$90.91$$

$$C_0 = \frac{C_1}{1 + r_1} = \text{PV}(C_1)$$

Check this—investing $90.91 at an interest rate of 10% will indeed return $100 next period:

$$10\% \approx \frac{\$100 - \$90.91}{\$90.91} = \frac{\$100}{\$90.91} - 1 \Leftrightarrow (1 + 10\%) \cdot \$90.91 \approx \$100$$

$$r_1 = \frac{C_1 - C_0}{C_0} = \frac{C_1}{C_0} - 1 \Leftrightarrow (1 + r_1) \cdot C_0 = C_1$$

Discounting translates future cash into today's equivalent.

This is the **present value formula**, which uses a division operation known as **discounting**. (The term "discounting" indicates that we are reducing a value, which is exactly what we are doing when we translate future cash into current cash.) If you wish, you can think of discounting—the conversion of a future cash flow amount into its equivalent present value amount—as the *reverse* of compounding.

Present value varies inversely with the cost of capital.

Thus, the present value (PV) of next year's \$100 is \$90.91—the value today of future cash flows. Let's say that this \$90.91 is what the project costs. If you can borrow or lend at the interest rate of 10% elsewhere, then you will be indifferent between receiving \$100 next year and receiving \$90.91 for your project today. In contrast, if the standard rate of return in the economy were 12%, your specific project would not be a good deal. The project's present value would be

$$\text{PV}(C_1) = \frac{\$100}{1 + 12\%} = \frac{\$100}{1.12} \approx \$89.29$$

$$C_0 = \frac{C_1}{1 + r_1} = \text{PV}(C_1)$$

which would be less than its cost of \$90.91. But if the standard economy-wide rate of return were 8%, the project would be a great deal. Today's present value of the project's future payoff would be

$$\text{PV}(C_1) = \frac{\$100}{1 + 8\%} = \frac{\$100}{1.08} \approx \$92.59$$

which would exceed the project's cost of \$90.91. It is the present value of the project, weighed against its cost, that should determine whether you should undertake a project today or avoid it. The present value is also the answer to the question, "How much would you have to save at current interest rates today if you wanted to have a specific amount of money next year?"

The PV formula with two periods.

Let's extend the time frame in our example. If the interest rate were 10% per period, what would \$100 in two periods be worth today? The value of the \$100 is then

$$\text{PV}(C_2) = \frac{\$100}{(1 + 10\%)^2} = \frac{\$100}{1.21} \approx \$82.64$$

$$\text{PV}(C_2) = \frac{C_2}{(1 + r)^2} = C_0 \qquad (2.2)$$

Note the 21%. In two periods, you could earn a rate of return of $(1 + 10\%) \cdot (1 + 10\%) - 1 = 1.1^2 - 1 = 21\%$ elsewhere, so this is your appropriate comparable rate of return.

The interest rate can be called the "cost of capital."

This discount rate—the rate of return, r, with which the project can be financed—is often called the **cost of capital**. It is the rate of return at which you can raise money elsewhere. In a perfect market, this cost of capital is also the **opportunity cost** that you bear if you fund your specific investment project instead of the alternative next-best investment elsewhere. Remember—you can invest your money at this opportunity rate in another project instead of this one. When these alternative projects in the economy elsewhere are better, your cost of capital is higher, and the value of your specific investment project with its specific cash flows is relatively lower. An investment that promises \$1,000 next year is worth less today if you can earn 50% rather than 5% elsewhere. A good rule is to always mentally add the word "opportunity" before "cost of capital"—it is always your **opportunity cost of capital**. (In this part of our book, I will just tell you what the economy-wide rate of return is—here 10%—for borrowing or investing. In later chapters, you will learn how this opportunity cost of capital [ahem "rate of return"] is determined.)

IMPORTANT

> Always think of the r in the present value denominator as your "opportunity" cost of capital. If you have great opportunities elsewhere, your projects have to be discounted at high discount rates. The discount rate, the cost of capital, and the required rate of return are really all just different names for the same thing.

The discount factor is a simple function of the cost of capital.

When you multiply a future cash flow by its appropriate **discount factor**, you end up with its present value. Looking at Formula 2.2, you can see that this discount factor is the quantity

$$\text{discount factor} \;=\; \left(\frac{1}{1+21\%}\right) \;\approx\; 0.8264$$

In other words, the discount factor translates 1 dollar in the future into the equivalent amount of dollars today. In the example, at a two-year 21% rate of return, a dollar in two years is worth about 83 cents today. Because interest rates are usually positive, discount factors are usually less than 1—a dollar in the future is worth less than a dollar today. (Sometimes, people call this the **discount rate**, but the discount rate is really $r_{0,t}$ if you are a stickler for accuracy.)

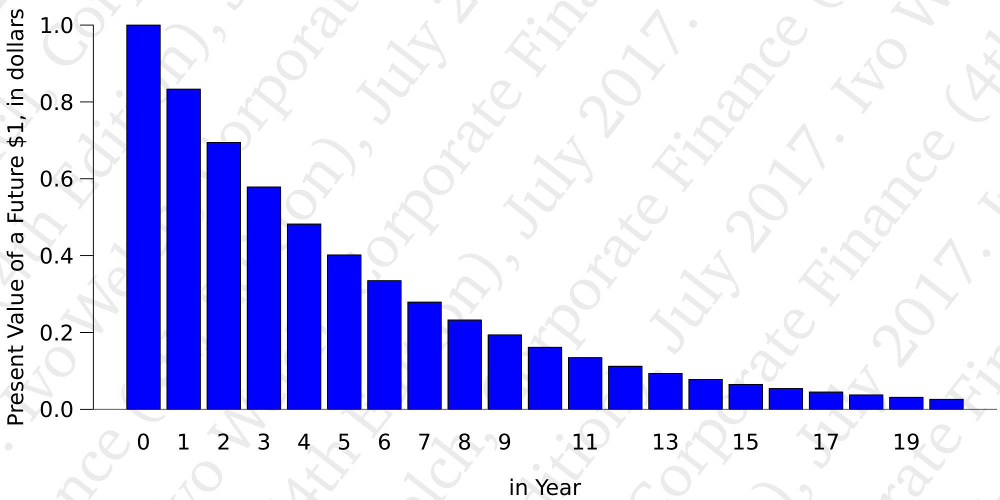


Exhibit 2.2: *Discounting over 20 Years at a Cost of Capital of 20% per Annum.* Each bar is $1/(1+20\%) \approx 83.3\%$ of the size of the bar to its left. After 20 years, the last bar is 0.026 in height. This means that $1 in 20 years is worth 2.6 cents in money today.

The discount rate is higher for years farther out, so the discount factor is lower.

Exhibit 2.2 shows how the discount factor declines when the cost of capital is 20% per annum. After about a decade, any dollar the project earns is worth less than 20 cents to you today. If you compare Exhibit 2.1 to Exhibit 2.2, you should notice how each is the "flip side" of the other.

IMPORTANT

The cornerstones of finance are the following formulas:

$$\text{Rate of Return:} \quad r_{0,t} = \frac{C_t - C_0}{C_0} = \frac{C_t}{C_0} - 1$$

Rearrange the formula to obtain the future value:

$$\text{Future Value:} \quad FV_t = C_t = C_0 \cdot (1 + r_t) = C_0 \cdot (1 + r)^t$$

The process of obtaining $r_{0,t}$ is called compounding, and it works through the "one-plus" formula:

$$\text{Compounding:} \quad \underbrace{(1 + r_{0,t})}_{\substack{\text{Total Holding} \\ \text{Rate of Return}}} = \underbrace{(1 + r)}_{\substack{\text{First Period} \\ \text{Rate of Return}}} \cdot \underbrace{(1 + r)}_{\substack{\text{Second Period} \\ \text{Rate of Return}}} \cdots \underbrace{(1 + r)}_{\substack{\text{Final Period} \\ \text{Rate of Return}}}$$

Rearrange the formula again to obtain the present value:

$$\text{Present Value:} \quad PV = C_0 = \frac{C_t}{(1 + r_{0,t})} = \frac{C_t}{(1 + r)^t}$$

The process of translating C_t into C_0—that is, the multiplication of a future cash flow by $1/(1 + r_{0,t})$—is called discounting. The discount factor is:

$$\text{Discount Factor:} \quad \frac{1}{(1 + r_{0,t})} = \frac{1}{(1 + r)^t}$$

It translates one dollar at time t into its equivalent value today.

Bonds' present values and the prevailing interest rates move in opposite directions.

Remember how bonds are different from savings accounts? The former is pinned down by its promised fixed future payments, while the latter pays whatever the daily interest rate is. This induces an important relationship between the value of bonds and the prevailing interest rates—*they move in opposite directions*. For example, if you have a bond that promises to pay $1,000 in one year, and the prevailing interest rate is 5%, the bond has a present value of $1,000/1.05 ≈ $952.38. If the prevailing interest rate suddenly increases to 6% (and thereby becomes your new opportunity cost of capital), the bond's present value becomes $1,000/1.06 ≈ $943.40. You lose $8.98, which is about 0.9% of your original $952.38 investment. The value of your fixed-bond payment in the future has gone down, because investors can now do better than your 5% by buying new bonds. They have better opportunities elsewhere in the economy. They can earn a rate of return of 6%, not just 5%, so if you wanted to sell your bond now, you would have to sell it at a discount to leave the next buyer a rate of return of 6%. If you had delayed your investment, the sudden change to 6% would have done nothing to your investment. On the other hand, if the prevailing interest rate suddenly drops to 4%, then your bond will be more valuable. Investors would be willing to pay $1,000/1.04 ≈ $961.54, which is an immediate $9.16 gain. The inverse relationship between prevailing interest rates and bond prices is general and worth noting.

IMPORTANT

The price and the implied rate of return on a bond with fixed payments move in opposite directions. When the price of the bond goes up, its implied rate of return goes down. When the price of the bond goes down, its implied rate of return goes up.

Q 2.27. A project with a cost of capital of 30% pays off $250. What should it cost today?

Q 2.28. A bond promises to pay $150 in 12 months. The annual true interest rate is 5% per annum. What is the bond's price today?

Q 2.29. A bond promises to pay $150 in 12 months. The bank quotes you an interest rate of 5% per annum, compounded daily. What is the bond's price today?

Q 2.30. If the cost of capital is 5% per annum, what is the discount factor for a cash flow in two years?

Q 2.31. Interpret the meaning of the discount factor.

Q 2.32. What are the units on rates of return, discount factors, future values, and present values?

Q 2.33. Would it be good or bad for you, in terms of the present value of your liabilities, if your opportunity cost of capital increased?

Q 2.34. The price of a bond that offers a safe promise of $100 in one year is $95. What is the implied interest rate? If the bond's interest rate suddenly jumped up by 150 basis points, what would the bond price be? How much would an investor gain/lose if she held the bond while the interest rate jumped up by these 150 basis points?

2.6 Net Present Value

Present values are alike and thus can be added, subtracted, compared, and so on.

An important advantage of present value is that all cash flows are translated into the same unit: cash today. To see this, say that a project generates $10 in one year and $8 in five years. You cannot add up these different future values to come up with $18—it would be like adding apples and oranges. However, if you translate both future cash flows into their present values, you *can* add them. For example, if the interest rate was 5% per annum (so $(1+5\%)^5 = (1+27.6\%)$ over 5 years), the present value of these two cash flows together would be

$$\text{PV}(\$10 \text{ in 1 year}) = \frac{\$10}{1.05} \approx \$9.52$$

$$\text{PV}(\$8 \text{ in 5 years}) = \frac{\$8}{1.05^5} \approx \$6.27$$

$$\text{PV}(C_t) = \frac{C_t}{(1+r)^t}$$

Therefore, the total value of the project's future cash flows *today* (at time 0) is $15.79.

The definition of NPV.

The **net present value (NPV)** of an investment is the present value of all its future cash flows minus the present value of its cost. It is really the same as present value, except that the word "net" upfront reminds you to add and subtract *all* cash flows, including the *upfront* investment outlay today. The NPV calculation method is always the same:

1. Translate all future cash flows into today's dollars.
2. Add them all up. This is the present value of all future cash flows.
3. Subtract the initial investment.

A basic use example

NPV is the most important method for determining the value of projects. It is a cornerstone of finance. Let's assume that you have to pay $12 to buy this particular project with its $10 and $8 cash flows. In this case, it is a positive NPV project, because

$$\text{NPV} = -\$12 + \frac{\$10}{1.05} + \frac{\$8}{1.05^5} \approx \$3.79$$

$$C_0 + \frac{C_1}{1+r_1} + \frac{C_5}{(1+r)^5} = \text{NPV}$$

(For convenience, we omit the 0 subscript for NPV, just as we did for PV.)

Think about what NPV means, and how it can be justified.

There are a number of ways to understand net present value.

- One way is to think of the NPV of \$3.79 as the difference between the market value of the future cash flows (\$15.79) and the project's cost (\$12)—this difference is the "value added."
- Another way to think of your project is to compare its cash flows to an equivalent set of bonds that exactly *replicates* them. In this instance, you would want to purchase a 1-year bond that promises \$10 next year. If you save \$9.52—at a 5% interest rate—you will receive \$10. Similarly, you could buy a 5-year bond that promises \$8 in year 5 for \$6.27. Together, these two bonds exactly replicate the project cash flows. The **law of one price** tells you that your project should be worth as much as this bond project—the cash flows are identical. You would have had to put away \$15.79 today to buy these bonds, but your project can deliver these cash flows at a cost of only \$12—much cheaper and thus better than your bond alternative.
- There is yet another way to think of NPV. It tells you how your project compares to the alternative opportunity of investing in the capital markets. These opportunities are expressed in the denominator through the discount factor. What would you get if you took your \$12 and invested it in the capital markets instead of in your project? Using the future value formula, you know that you could earn a 5% rate of return from now to next year, and 27.6% from now to 5 years. Your \$12 would grow into \$12.60 by next year. You could take out the same \$10 cash flow that your project gives you and be left with \$2.60 for reinvestment. Over the next 4 years, at the 5% interest rate, this \$2.60 would grow into \$3.16. But your project would do better for you, giving you \$8. Thus, your project achieves a higher rate of return than the capital markets alternative.

Yet another way to justify NPV: opportunity cost.

The correct capital budgeting rule: Take all positive NPV projects.

The conclusion of this argument is not only the simplest but also the best capital budgeting rule: If the NPV is positive, as it is for our \$3.79 project, you should take the project. If it is negative, you should reject the project. If it is zero, it does not matter.

IMPORTANT

- The NPV formula is

$$\begin{aligned}
\text{NPV} &= C_0 + \text{PV}(C_1) + \text{PV}(C_2) + \text{PV}(C_3) + \text{PV}(C_4) + \cdots \\
&= C_0 + \frac{C_1}{1+r_1} + \frac{C_2}{1+r_2} + \frac{C_3}{1+r_3} + \frac{C_4}{1+r_4} + \cdots \\
&= C_0 + \frac{C_1}{(1+r)} + \frac{C_2}{(1+r)^2} + \frac{C_3}{(1+r)^3} + \frac{C_4}{(1+r)^4} + \cdots
\end{aligned}$$

  The subscripts are time indexes, C_t is the net cash flow at time t (positive for inflows, negative for outflows), and r_t is the relevant interest rate for investments from now to time t. With constant interest rates, $r_t = (1+r)^t - 1$.
- The **NPV capital budgeting rule** states that you should accept projects with a positive NPV and reject those with a negative NPV.
- Taking positive NPV projects increases the value of the firm. Taking negative NPV projects decreases the value of the firm.
- NPV is definitively the best method for capital budgeting—the process by which you should accept or reject projects.

The NPV formula is so important that you must memorize it.

Let's work a project NPV example.

Let's work another NPV example. A project costs $900 today, yields $200/year for two years, then $400/year for two years, and finally requires a cleanup expense of $100. The prevailing interest rate is 5% per annum. These cash flows are summarized in Exhibit 2.3. Should you take this project?

First, determine your multiyear costs of capital.

1. You need to determine the cost of capital for tying up money for one year, two years, three years, and so on. The compounding formula is

$$(1 + r_t) = (1 + r)^t = (1.05)^t = 1.05^t$$

So for money right now, the cost of capital r_0 is $1.05^0 - 1 = 0$; for money in one year, r_1 is $1.05^1 - 1 = 5\%$; for money in two years, r_2 is $1.05^2 - 1 = 10.25\%$. And so on.

2. You need to translate the cost of capital into discount factors. Recall that these are 1 divided by 1 plus your cost of capital. A dollar in one year is worth $1/(1+5\%) = 1/1.05 \approx 0.9524$ dollars today. A dollar in two years is worth $1/(1+5\%)^2 = 1/1.05^2 \approx 0.9070$. And so on.

3. You can now translate the future cash flows into their present value equivalents by multiplying the payoffs by their appropriate discount factors. For example, the $200 cash flow at time 1 is worth about $0.9524 \cdot \$200 \approx \$190.48$.

4. Because present values are additive, you then sum up all the terms to compute the overall net present value. Make sure you include the original upfront cost as a negative.

➤ $68.14 or $68.15?: Rounding Error, Pg.20.

Consequently, the project NPV is about $68.15. Because $68.15 is a positive value, you should take this project.

		Interest Rate			
Time	Project Cash Flow	Annualized	Discount Holding	Present Factor	Value
t	C_t	r	r_t	$\frac{1}{(1+r)^t}$	$PV(C_t)$
Today 0	–$900	5.00%	0.00%	1.0000	–$900.00
Year +1	+$200	5.00%	5.00%	0.9524	+$190.48
Year +2	+$200	5.00%	10.25%	0.9070	+$181.41
Year +3	+$400	5.00%	15.76%	0.8638	+$345.54
Year +4	+$400	5.00%	21.55%	0.8227	+$329.08
Year +5	–$100	5.00%	27.63%	0.7835	–$78.35
			Net Present Value (Sum):		$68.15

Exhibit 2.3: *Hypothetical Project Cash Flow Table.* As a manager, you must provide estimates of your project cash flows. The appropriate interest rate (also called cost of capital in this context) is provided to you by the opportunity cost of your investors—determined by the supply and demand for capital in the broader economy, where your investors can invest their capital instead. The "Project Cash Flow" and the left interest rate column are the two input columns. The remaining columns are computed from these inputs. The goal is to calculate the final column.

However, if the upfront expense was $1,000 instead of $900, the NPV would be negative (–$31.84), and you would be better off investing the money into the appropriate sequence of bonds from which the discount factors were computed. In this case, you should have rejected the project.

If the upfront cost was higher, you should not take the project.

Q 2.35. Work out the present value of your tuition payments for the next two years. Assume that the tuition is $30,000 per year, payable at the start of the year. Your first tuition payment will occur in 6 months, and your second tuition payment will occur in 18 months. You can borrow capital at an effective interest rate of 6% per annum.

Q 2.36. Write down the NPV formula from memory.

Q 2.37. What is the NPV capital budgeting rule?

Q 2.38. Determine the NPV of the project in Exhibit 2.3, if the per-period interest rate were 8% per year, not 5%. Should you take this project?

Q 2.39. You are considering moving into for a building for three years, for which you have to make one payment now, one in a year, and a final one in two years.

1. Would you rather have a lease, paying $1,000,000 upfront, then $500,000 each in the following two years; or would you rather pay $700,000 rent each year?
2. If the interest rate is 10%, what equal payment amount (rather than $700,000) would leave you indifferent? (This is also called the equivalent annual cost (EAC).)

Q 2.40. Use a spreadsheet to answer the following question: Car dealer A offers a car for $2,200 upfront (first payment), followed by $200 lease payments over the next 23 months. Car dealer B offers the same lease at a flat $300 per month (i.e., your first upfront payment is $300). Which lease do you prefer if the interest rate is 0.5% per month?

Application: Are Faster-Growing Firms Better Bargains?

Let's work another NPV problem, applying to companies overall. Does it make more sense to invest in companies that are growing quickly rather than slowly? If you wish, you can think of this question loosely as asking whether you should buy stock of a fast-growing company like Google or stock of a slow-growing company like Procter & Gamble. Actually, you do not even have to calculate anything. In a perfect market, the answer is always that every publicly traded investment comes for a fair price. Thus, the choice does not matter. Whether a company is growing quickly or slowly is already incorporated in the firm's price today, which is just the present value of the firm's cash flows that will accrue to the owners. Therefore, neither is the better deal. Yet, because finance is so much fun, we will ignore this little nuisance and work out the details anyway.

The firm's price should incorporate the firm's attributes.

For example, say company "Grow" (G) will produce over the next 3 years

Should you invest in a fast-grower or a slow-grower?

$$G_1 = \$100 \qquad G_2 = \$150 \qquad G_3 = \$250$$

and company "Shrink" (S) will produce

$$S_1 = \$100 \qquad S_2 = \$90 \qquad S_3 = \$80$$

Is G not a better company to buy than S?

There is no uncertainty involved, and both firms face the same cost of capital of 10% per annum. The price of G today is its present value (PV)

Let's find out: Compute the values.

$$PV(G) = \frac{\$100}{1.1^1} + \frac{\$150}{1.1^2} + \frac{\$250}{1.1^3} \approx \$402.70 \quad (2.3)$$

and the price of S today is

$$PV(S) = \frac{\$100}{1.1^1} + \frac{\$90}{1.1^2} + \frac{\$80}{1.1^3} \approx \$225.39$$

Your investment dollar grows at the same 10% rate. Your investment's growth rate is disconnected from the cash flow growth rate.

What is your rate of return from this year to next year? If you invest in G, then next year you will have \$100 cash and own a company with \$150 and \$250 cash flows coming up. G's value at time 1 (so PV now has subscript 1 instead of the usually omitted 0) will thus be

$$PV_1(G) = \$100 + \frac{\$150}{1.1^1} + \frac{\$250}{1.1^2} \approx \$442.98$$

Your investment will have earned a rate of return of $\$442.98/\$402.70 - 1 \approx 10\%$. If you invest instead in S, then next year you will receive \$100 cash and own a company with "only" \$90 and \$80 cash flows coming up. S's value will thus be

$$PV_1(S) = \$100 + \frac{\$90}{1.1^1} + \frac{\$80}{1.1^2} \approx \$247.93$$

Your investment will have earned a rate of return of $\$247.93/\$225.39 - 1 \approx 10\%$. In either case, you will earn the fair rate of return of 10% from this year to next year. Whether cash flows are growing at a rate of +50%, -10%, +237.5%, or -92% is irrelevant: *The firms' market prices today already reflect their future growth rates*. There is no necessary connection between the growth rate of the underlying project cash flows or earnings and the growth rate of your investment money (i.e., your expected rate of return).

Any sudden wealth gains would accrue to existing shareholders, not to new investors.

Make sure you understand the thought experiment here: This statement that higher-growth firms do not necessarily earn a higher rate of return does not mean that a firm in which managers succeed in increasing the future cash flows at no extra investment cost will not be worth more. Such firms will indeed be worth more, and the current owners will benefit from the rise in future cash flows, but this will also be reflected immediately in the price at which you, an outsider, can buy this firm. This is an important corollary worth repeating. If General Electric has just won a large defense contract (like the equivalent of a lottery), shouldn't you purchase GE stock to participate in the windfall? Or if Wal-Mart managers do a great job and have put together a great firm, shouldn't you purchase Wal-Mart stock to participate in this windfall? The answer is that you cannot. The old shareholders of Wal-Mart are no dummies. They know the capabilities of Wal-Mart and how it will translate into cash flows. Why should they give you, a potential new shareholder, a special bargain for something to which you contributed nothing? Just providing more investment funds is not a big contribution—after all, there are millions of other investors equally willing to provide funds at the appropriate right price. It is competition—among investors for providing funds and among firms for obtaining funds—that determines the expected rate of return that investors receive and the cost of capital that firms pay. There is actually a more general lesson here. Economics tells you that you must have a scarce resource if you want to earn above-normal profits. Whatever is abundant and/or provided by many competitors will not be a tremendously profitable business.

An even more general lesson.

An even more general version of the question in this section (whether fast-growing or slow-growing firms are better investments) is whether good companies are better investments than bad companies. Many novices will answer that it is better to buy a good company. But you should immediately realize that the answer must depend on the price. Would you really want to buy a great company if its cost was twice its value? And would you really not want to buy a lousy company if you could buy it for half its value? For an investment, whether a company is a well-run purveyor of fine perfume or a poorly-run purveyor of fine manure does not matter by itself. What matters is only the company price relative to the future company cash flows that you will receive.

Q 2.41. Assume that company G pays no interim dividends, so you receive \$536 at the end of the project. What is G's market value at time 1, 2, and 3? What is your rate of return in each year? Assume that the cost of capital is still 10%.

Q 2.42. Assume that company G pays out the full cash flows (refer to the text example) in earnings each period. What is G's market value after the payout at time 1, 2, and 3? What is your rate of return in each year?

Q 2.43. One month ago, a firm suffered a large court award against it that will force it to pay compensatory damages of \$100 million next January 1. Are shares in this firm a bad buy until January 2?

Summary

This chapter covered the following major points:

- A perfect market assumes no taxes, no transaction costs, no opinion differences, and the presence of many buyers and sellers.
- A bond is a claim that promises to pay an amount of money in the future. Buying a bond is extending a loan. Issuing a bond is borrowing. Bond values are determined by their future payoffs.
- One hundred basis points are equal to 1%.
- The time value of money means that 1 dollar today is worth more than 1 dollar tomorrow because of the interest that it can earn.
- Returns must not be averaged, but compounded over time.
- Interest rate quotes are *not* interest rates. For example, stated annual rates are usually not the effective annual rates that your money will earn in the bank. If in doubt, ask!
- The discounted present value (PV) translates future cash values into present cash values. The net present value (NPV) is the sum of all present values of a project, including the investment cost (usually, a negative upfront cash flow today).
- The values of bonds and interest rates move in opposite directions. A sudden increase in the prevailing economy-wide interest rate decreases the present value of a bond's future payouts and therefore decreases today's price of the bond. Conversely, a sudden decrease in the prevailing economy-wide interest rate increases the present value of a bond's future payouts and therefore increases today's price of the bond.
- The NPV formula can be written as

$$\begin{aligned} NPV &= C_0 + \frac{C_1}{1+r_1} + \frac{C_2}{1+r_2} + \cdots \\ &= C_0 + \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + \cdots \end{aligned}$$

  In this context, r is called the discount rate or cost of capital, and $1/(1+r)$ is called the discount factor.
- The net present value capital budgeting rule states that you should accept projects with a positive NPV and reject projects with a negative NPV.
- In a perfect market, firms are worth the present value of their assets. Whether firms grow quickly or slowly does not make them more or less attractive investments in a perfect market, because their prices always already reflect the present value of future cash flows.
- In a perfect market, the gains from sudden surprises accrue to old owners, not new capital providers, because old owners have no reason to want to share the spoils.

Keywords

Answers

Q 2.1 The four perfect market assumptions are no taxes, no transaction costs, no differences in opinions, and no large buyers or sellers.

Q 2.2 A savings deposit is an investment in a series of short-term bonds.

Q 2.3 $r = (\$1{,}050 - \$1{,}000)/\$1{,}000 = 5\%$

Q 2.4 $r = \dfrac{\$25}{\$1{,}000} = 2.5\%$

Q 2.5 Yes, $10 = 1{,}000\%$.

Q 2.6 The dividend yield would be $\$1/\$40 = 2.5\%$, the capital gain would be $\$45 - \$40 = \$5$, so that its capital gain yield would be $\$5/\$40 = 12.5\%$, and the total rate of return would be $(\$46 - \$40)/\$40 = 15\%$.

Q 2.7 $1\% = 100$ basis points, so an increase of 3% is 300 basis points.

Q 2.8 20 basis points are 0.2%, so the interest rate declined from 10.0% to 9.8%.

Q 2.9 $r = 30\% = (x - \$250)/\$250 \implies x = 1.3 \cdot \$250 = \$325$

Q 2.10 $1.20^5 - 1 \approx 148.83\%$

Q 2.11 $\$2{,}000 \cdot 1.25^{15} \approx \$56{,}843.42$

Q 2.12 The total holding rate of return is $1.05^{20} - 1 \approx 165.33\%$, so you would end up with $\$200 \cdot (1 + 165.33\%) \approx \$530.66$.

Q 2.13 Losing one-third is a rate of return of –33%. To find the holding rate of return, compute $[1 + (-1/3)]^5 - 1 \approx -86.83\%$. About $(1 - 86.83\%) \cdot \$20{,}000 \approx \$2{,}633.74$ remains.

Q 2.14 $(1 + 100\%)^{1/5} - 1 \approx 14.87\%$

Q 2.15 $(1 + r_{0.25})^4 = (1 + r_1)$. Thus, $r_{0.25} = \sqrt[4]{1 + r_1} - 1 = 1.5^{1/4} - 1 \approx 10.67\%$.

Q 2.16 $r_2 = (1 + r_{0,1}) \cdot (1 + r_{1,2}) - 1 = 1.05 \cdot 1.05 - 1 = 10.25\%$

Q 2.17 $r_{10} = (1 + r_1)^{10} - 1 = 1.05^{10} - 1 \approx 62.89\%$

Q 2.18 $r_{100} = (1 + r_1)^{100} - 1 = 1.05^{100} - 1 = 130.5 \approx 13{,}050\%$. In words, this is about 130 times the initial investment, and about 26 times more than the 500% (5 times the initial investment).

Q 2.19 Tripling is equivalent to earning a rate of return of 200%. Therefore, solve $(1 + 6\%)^x = (1 + 200\%)$, or $x \cdot \log(1.06) = \log(3.00)$ or $x = \log(3.00)/\log(1.06) \approx 18.85$ years.

Q 2.20 $(1 + r)^{365.25} = 1.12$. Therefore, $1.12^{(1/365.25)} - 1 \approx 0.000310 = 0.0310\% \approx 3.10\text{bp/day}$.

Q 2.21 The bank pays $12\%/365.25 \approx 3.28\text{bp/day}$.

Q 2.22 This question demonstrates a nuisance problem that is pervasive in this book: calculations often have rounding error, especially when intermediate results are shown. The following three routes are logically the same, but the precise number differs based on when and where you round:

- Based on 365.25 days per year (which is incidentally itself rounded from the more exact 365.2422 days), the true daily interest rate is 0.00031032517117.... If you use full precision in your calculations, your weekly interest comes to $1.00031032517117\ldots^7 - 1 \approx 0.002174300\ldots$.
- If you round the true daily interest rate to 0.00031, your weekly interest comes to $1.00031\ldots^7 - 1 \approx 0.002172\ldots$.
- Based on 52.2 weeks per year (itself rounded from 52.177 weeks), you could have computed $r = (1 + 12\%)^{(1/52.2)} - 1 \approx 0.002173406\ldots$.

In the \$1,000 case, all three methods give you the same answer of \$1,002.17. In the \$100,000 case, you would have ended up with slightly different numbers based on your route of calculation. All three methods would have been acceptably correct.

In any case, don't blame this book or yourself for small discrepancies in calculations.

Q 2.23 With 12% in nominal APR interest *quoted*, you earn $12\%/365 \approx 0.032877\%$ per day. Therefore, the weekly rate of return is $(1 + 0.032877\%)^7 - 1 \approx 0.23036\%$. Your \$1,000 will grow into \$1,002.30. Note that you end up with more money when the 12% is the quoted rate than when it is the effective rate.

Q 2.24 With 12% in nominal APR interest *quoted*, you earn 12%/365 ≈ 0.032877% per day. Therefore, the annual rate of return is $(1 + 0.032877\%)^{365} - 1 \approx 12.747462\%$. Your \$1,000 will grow into \$1,127.47.

Q 2.25 The bank quote of 6% means that it will pay an interest rate of 6%/365 ≈ 0.0164384% per day. This earns an actual interest rate of $(1 + 0.0164384\%)^{365} - 1 \approx 6.18\%$ per annum. Therefore, each invested \$100 grows to \$106.18, thus earning \$6.18 over the year.

Q 2.26 The bank quote of 8% means that you will have to pay an interest rate of 8%/12 ≈ 0.667% per month. This earns an actual interest rate of $(1 + 0.667\%)^{12} - 1 \approx 8.30\%$ per annum. You will have to pay \$108.30 in repayment for every \$100 you borrowed.

Q 2.27 $r = 30\% = (\$250 - x)/x$. Thus, $x = \$250/1.30 \approx$ \$192.31.

Q 2.28 $\$150/(1.05) \approx \$142.86$

Q 2.29 $\$150/[1 + (5\%/365)]^{365} \approx \$142.68$

Q 2.30 $1/[(1.05) \cdot (1.05)] \approx 0.9070$

Q 2.31 It is today's value in dollars for 1 future dollar, that is, at a specific point in time in the future.

Q 2.32 The rate of return and additional factors are unit-less. The latter two are in dollars (though the former is dollars in the future, while the latter is dollars today).

Q 2.33 Good. Your future payments would be worth less in today's money.

Q 2.34 The original interest rate is \$100/\$95 – 1 ≈ 5.26%. Increasing the interest rate by 150 basis points is 6.76%. This means that the price should be \$100/(1.0676) ≈ \$93.67. A price change from \$95 to \$93.67 is a rate of return of \$93.67/\$95 – 1 ≈ –1.40%.

Q 2.35 The first tuition payment is worth $\$30{,}000/(1.06)^{1/2} \approx$ \$29,139. The second tuition payment is worth $\$30{,}000/(1.06)^{3/2} \approx$ \$27,489. Thus, the total present value is \$56,628.

Q 2.36 If you cannot write down the NPV formula by heart, do not go on until you have it memorized.

Q 2.37 Accept if NPV is positive. Reject if NPV is negative.

Q 2.38 $-\$900 + \$200/(1.08)^1 + \$200/(1.08)^2 + \$400/(1.08)^3 + \$400/(1.08)^4 - \$100/(1.08)^5 \approx \$0.14$. The NPV is positive. Therefore this is a worthwhile project that you should accept.

Q 2.39 For the 3-years:

1. Your rent-vs-lease preference depends on the interest rate. If the interest rate is zero, then you would prefer the \$2 million sum-total lease payments to the \$2.1 million sum-total rent payments. If the prevailing interest rate is less than 21.5%, it is better to lease. If it is more than 21.5%, you prefer the rent. For example, if it is 40%, the net present cost of the lease is \$1.612 million, while the net present cost of the rent is \$1.557 million.
2. At a 10% interest rate, the total net present cost of the lease is $\$1 + \$0.5/1.1 + \$0.5/1.1^2 \approx \$1.868$ million. An equivalent rent contract must solve

$$x + \frac{x}{1.1} + \frac{x}{1.1^2} = \$1.868$$

Multiply by $1.1^2 = 1.21$

$$1.21 \cdot x + 1.1 \cdot x + x = \$1.868 \cdot 1.21$$
$$\Leftrightarrow x \cdot (1.21 + 1.1 + 1) = \$2{,}260.28$$

Therefore, the equivalent rental cost would be $x \approx \$682.864$.

Q 2.40 Lease A has an NPV of –\$6,535. Lease B has an NPV of –\$6,803. Therefore, lease A is cheaper.

Q 2.41 For easier naming, let's use a specific year. Pretend it is the year 2000 now, and call 2000 your year 0. (Coincidence that the final digit is the same?!) The firm's present value in 2000 is $\$536/1.10^3 \approx \$402.70$—but you already knew this. If you buy this company, its value in 2001 depends on a cash flow stream that is \$0 in 2001, \$0 in year 2002, and \$536 in year 2003. It will be worth $\$536/1.10^2 \approx \$442.98$ in 2001. In 2002, your firm will be worth \$536/1.10 ≈ \$487.27. Finally, in 2003, it will be worth \$536. Each year, you expect to earn 10%, which you can compute from the four firm values.

Q 2.42 Again, call 2000 your year 0. The firm's present value in 2000 is based on dividends of \$100, \$150, and \$250 in the next three years. The firm value in 2000 is the \$402.70 from Page 30. The firm value in 2001 was also worked out to be \$442.98, but you immediately receive \$100 in cash, so the firm is worth only \$442.98 – \$100 = \$342.98. As an investor, you would have earned a rate of return of \$442.98/\$402.70 – 1 ≈ 10%. The firm value in 2002 is $PV_2(G) = \$250/1.1 \approx \$227.27$, but you will also receive \$150 in cash, for a total firm-related wealth of \$377.27. In addition, you will have the \$100 from 2001, which would have grown to \$110—for a total wealth of \$487.27. Thus, starting with wealth of \$442.98 and ending up with wealth of \$487.27, you would have earned a rate of return of \$487.27/\$442.98 – 1 ≈ 10%. A similar computation shows that you will earn 10% from 2002 (\$487.27) to 2003 (\$536.00).

Q 2.43 No! The market price will have already taken the compensatory damages into account in the share price a month ago, just after the information had become public.

End of Chapter Problems

Q 2.44. What is a perfect market? What were the assumptions made in this chapter that were not part of the perfect market scenario?

Q 2.45. In the text, I assumed you received the dividend at the end of the period. In the real world, if you received the dividend at the beginning of the period instead of the end of the period, could this change your effective rate of return? Why?

Q 2.46. Your stock costs $100 today, pays $5 in dividends at the end of the period, and then sells for $98. What is your rate of return?

Q 2.47. What is the difference between a bond and a loan?

Q 2.48. Assume an interest rate of 10% per year. How much would you lose over 5 years if you had to give up interest on the interest—that is, if you received 50% instead of compounded interest?

Q 2.49. The interest rate has just increased from 6% to 8%. How many basis points is this?

Q 2.50. Over 20 years, would you prefer 10% per annum, with interest compounding, or 15% per annum but without interest compounding? (That is, you receive the interest, but it is put into an account that earns no interest, which is what we call simple interest.)

Q 2.51. A project returned +30%, then –30%. Thus, its arithmetic average rate of return was 0%. If you invested $25,000, how much did you end up with? Is your rate of return positive or negative? How would your overall rate of return have been different if you first earned –30% and then +30%?

Q 2.52. A project returned +50%, then –40%. Thus, its arithmetic average rate of return was $(50\% + [-40\%])/2 = +5\%$. Is your rate of return positive or negative?

Q 2.53. An investment for $50,000 earns a rate of return of 1% in each month of a full year. How much money will you have at the end of the year?

Q 2.54. There is always disagreement about what stocks are good buys. A typical disagreement is whether a particular stock is likely to offer, say, a 10% (pessimistic) or a 20% (optimistic) annualized rate of return. For a $30 stock today, what does the difference in belief between these two opinions mean for the expected stock price from today to tomorrow? (Assume that there are 365 days in the year. Reflect on your answer for a moment—a $30 stock typically moves about ±$1 on a typical day. This unexplainable up-and-down volatility is often called noise. How big is the average move compared to the noise?)

Q 2.55. If the interest rate is 5% per annum, how long will it take to double your money? How long will it take to triple it?

Q 2.56. If the interest rate is 8% per annum, how long will it take to double your money?

Q 2.57. From Fibonacci's *Liber Abaci*, written in the year 1202: "A certain man gave 1 denaro at interest so that in 5 years he must receive double the denari, and in another 5, he must have double 2 of the denari and thus forever. How many denari from this 1 denaro must he have in 100 years?"

Q 2.58. A bank quotes you an annual loan interest rate of 14%, daily compounding, on your credit card. If you charge $15,000 at the beginning of the year, how much will you have to repay at the end of the year?

Q 2.59. Go to the website of a bank of your choice. What kind of quote does your bank post for a CD, and what kind of quote does your bank post for a mortgage? Why?

Q 2.60. What is the 1-year discount factor if the interest rate is 33.33%?

Q 2.61. You can choose between the following rent payments:

a A lump sum cash payment of $100,000;
b 10 annual payments of $12,000 each, the first occurring immediately;
c 120 monthly payments of $1,200 each, the first occurring immediately. (Friendly suggestion: This is a lot easier to calculate on a computer spreadsheet.)

Now choose among them:

1. Which rental payment scheme would you choose if the interest rate was an effective 5% per year?
2. Spreadsheet question: At what interest rate would you be indifferent between the first and the second choice above? (Hint: Graph the NPV of the second project as a function of the interest rate.)

Q 2.62. A project has cash flows of $15,000, $10,000, and $5,000 in 1, 2, and 3 years, respectively. If the prevailing interest rate is 15%, would you buy the project if it costs $25,000?

Q 2.63. Consider the same project that costs $25,000 with cash flows of $15,000, $10,000, and $5,000. At what prevailing interest rate would this project be profitable? Try different interest rates, and plot the NPV on the y-axis, and the interest rate on the x-axis.

Q 2.64. Assume you are 25 years old. The IAW insurance company is offering you the following retirement contract (called an *annuity*): Contribute $2,000 per year for the next 40 years. When you reach 65 years of age, you will receive $30,000 per year for as long as you live. Assume that you believe that the chance that you will die is 10% per year after you will have reached 65 years of age. In other words, you will receive the first payment with probability 90%, the second payment with probability 81%, and so on. If the prevailing interest rate is 5% per year, all payments occur at year-end, and it is now January 1, is this annuity a good deal? (Use a spreadsheet.)

Q 2.65. A project has the following cash flows in periods 1 through 4: –$200, +$200, –$200, +$200. If the prevailing interest rate is 3%, would you accept this project if you were offered an upfront payment of $10 to do so?

Q 2.66. On January 1, 2016, Intel Corp's stock traded for $33.99. In 2012, it paid $0.21/quarter in dividends, then $0.225 in dividends until 2015 when it increased to $0.24, and finally to $0.26 in 2016. Assume Intel will pay $0.25/quarter in 2017. Further assume that the prevailing interest rate is 0.5% per quarter (i.e., 2.015% per annum). If you buy Intel stock on January 1, 2016, at what price would you have to be able to sell Intel stock at the end of 2017 in order to break even?

Q 2.67. If the interest rate is 5% per annum, what would be the equivalent annual cost (see Question 2.39) of a $2,000 lease payment upfront, followed by $800 for three more years?

Q 2.68. Assume that you are a real estate broker with an exclusive contract—the condo association rules state that everyone selling their condominiums must go through you or a broker designated by you. A typical condo costs $500,000 today and sells again every 5 years. Assume the first sale will happen in 5 years. This will last for 50 years, and then all bets are off. Your commission will be 3%. Condos appreciate in value at a rate of 2% per year. The interest rate is 10% per annum.

1. What is the value of this exclusivity rule for one condo? In other words, at what price should you be willing to sell the privilege of being the exclusive representation for one condo to another broker?
2. If free Internet advertising was equally effective and if it could replace all real-estate agents so that buyers' and sellers' agents would no longer earn the traditional 6% (3% each), what would happen to the value gain of the condo?

Q 2.69. The prevailing discount rate is 15% per annum. Firms live for three years. Firm F's cash flows start with $500 in year 1 and grow at 20% per annum for two years. Firm S's cash flows also start with $500 in year 1 but shrink at 20% per annum for two years. What are the prices of these two firms? Which one is the better "buy"?