10

The Capital Asset Pricing Model

What cost of capital does your project have to offer? The last chapter explained how you can determine the expected rate of return if your project is like other assets — such as Treasuries, the stock market, or some other traded financial assets.

But what about projects that are not easily decomposed into obvious combinations? How would you judge how much of other assets you would need to mimic your project? And which other assets should you then choose as your benchmarking portfolios?

This is the domain of the Capital Asset Pricing Model (CAPM). It hypothesizes that the only two important aspects are the rate of return on the risk-free rate and the stock-market. If you are willing to buy into its premise, the CAPM gives you a beautifully simple and intuitive formula that relates how much reward your investment project has to offer in order to compensate your investors for its risk. The risk is the market beta. By assumption, the formula works for *any* kind of project. You can then use its costs of capital in your NPV calculations.

We will first briefly review what you already know. Then you will learn all about the CAPM. And you will get to apply it.

And then I will have to deliver the bad news: although the CAPM remains the dominant model in practice (and in job interviews), not only is it often poorly applied, but it has also performed so poorly (and increasingly poorly over the last few decades) that finance researchers no longer believe that it is a good model for predicting which stocks have high and which stocks have low expected rates of return. Its empirical validity was miserable even under the most sympathetic assumptions. On average, you would have been better off simply ignoring it — or even assuming the opposite. It's our collective conundrum. Welcome to the club.

Wait a little — I will put it all in perspective for you.

Funny Youtube Video from your's truly: "CAPM crunched"

Ivo Welch, Corporate Finance (5th Edition, Aug 2022), book.ivo-welch.info.

10.1 What You Already Know

We want a more universal model comparing all sorts of investments.

We are still going after finding the answer to the same central question — what should we, as corporate managers, consider to be a good (opportunity) cost of capital for the projects we are considering deploying? We already know one good measure from the previous chapter: We can compare the price of our own project to those of benchmark projects that are already traded in competitive public markets. But there can be a problem. It is not always easy to find good obvious benchmarks for all our projects. We would love a model that is a bit less specific and a bit more generic in its prescriptions. It should tell us about the cost of capital even if we do not have a good benchmark. That is, we would love a model that gave us the equivalent of a universal benchmark.

The big new assumption: investors are risk-averse.

To get such a model, we have to push on some assumptions. We will assume that investors are risk-averse. From the previous chapters, you know that if expected rates of return were the same, investors would prefer short-term low-risk project cash flows (like overnight Treasuries) to long-term unsafe project cash flows (like the stock market). We also assume that investors are smart, so they diversify their portfolios to reduce their risk exposures. The types of risk that investors consider noxious can only be the parts that they cannot wash out by diversification and that remain left over even when all assets are just tiny parts of their large overall portfolios.

Perfect markets: Investors compete for good deals.

We also continue to lean on our perfect-market assumptions just as we have in recent chapters. Thus, not being dummies, investors collectively snatch up the best projects — those that have low risk and high expected rates of return. In fact, anyone contemplating selling a project with more reward than it deserves would be needlessly giving away bargains that would attract a gazillion bidders. Anyone contemplating selling projects with too unfavorable risk contributions for their rewards would not receive a single offer. There is really only one correct choice of price. Consequently, what investors purchase in the real world at the correct prices must be subject to some trade-off: Projects that drive up investors' overall portfolio risk must offer higher expected rates of return.

We would love an integrated "beautiful" perspective.

These simplifications will leave you with a consistent conceptual framework: External investors dislike overall portfolio risk (standard deviation) and like reward (expected returns). They care about their overall financial investments portfolio. They are smart, so they are diversified. With a few more assumptions, this means most likely that they are holding something akin to the overall stock-market portfolio for risk and something like risk-free Treasuries for safety.

Assume what investors like, and you can figure out how they judge your project.

Given this CAPM conceptual framework about how external investors want to invest, we can skip back to the perspective of the corporate manager. We know that our assumed model says investors will judge our project in terms of how it is "more like stocks" rather than how it is "more like Treasuries." The measure that gives us the relative type is the "market beta." The CAPM puts this all together — the risk-free rate of return (like Treasuries), the expected rate of return on risky assets (like stocks), and the market-beta that measures how much of each.

Oh, and they are assumed to care only about risk and reward.

In the "risk-reward game," we measure investors' reward are their portfolios' expected rate of return. Investors' risk is their *overall* portfolio risk, *not* your project's *own* standard-deviation risk. Your own project's contribution to investors' overall portfolio risk is then best measured by the market beta of your project.

A good way to think of beta is as a measure of your project's "noxiousness." A project that decreases in value when the market decreases in value (and increases when it increases) has a positive market beta. It's noxious — investors don't like it. A project that increases in value when the market decreases in value, and vice-versa, has a negative market beta. It's less noxious — investors like it more. That is, projects with lower market betas help investors (who already otherwise hold market-like portfolios) suffer less overall portfolio risk.

Beta is not loved — either by struggling students or by investors.

Another way to think of non-synchronicity is that oranges in winter are more valuable than oranges in summer (when oranges are plentiful). Non-synchronicity is a desirable attribute, and beta is a measure of synchronicity.

10.2 The Capital Asset Pricing Model (CAPM)

The **capital asset pricing model** (**CAPM**) gives you a formula that relates the appropriate expected rate of return (cost of capital) for your project — if you feed it your project's single relevant risk characteristics (the market beta), the risk-free rate of return and the expected return on the stock-market (or equivalently the equity premium). Note that you also had to provide the expected rate of return on the stock-market in the previous benchmarking chapter. It is the most uncertain and thus difficult input into the CAPM.

The CAPM gives you the cost of capital if you give it the risk-free rate, the expected rate of return on the market, and your project's market beta.

The CAPM states that an investment's cost of capital is lower when it offers better diversification benefits for investors who hold the overall market portfolio — less required reward for less risk contribution. Market beta is the model's measure of risk contribution. Projects contributing more risk (market beta) require higher expected rates of return (for you to want them on behalf of your investors); projects contributing less risk require lower expected rates of return. According to the CAPM, nothing but the risk-free rate, the expected equity premium, and the market beta matters. No other financial assets need to be investigated to judge your project.

More noxious assets (with higher betas) require paying higher expected rates of return.

Important

To estimate the required expected rate of return for a project or firm — that is, the cost of capital — according to the CAPM, you need three inputs:

1. The risk-free rate of return, r_F.
2. The expected rate of return on the overall market, $\mathsf{E}(r_M)$ — or, alternatively, the equity premium $\mathsf{E}(r_M) - r_F$.
3. The project's beta with respect to the market, β_i.

The CAPM formula is

$$\mathsf{E}(r_i) = r_F + [\mathsf{E}(r_M) - r_F] \cdot \beta_i$$

where i is the name of your project and $\mathsf{E}(r_i)$ is your project's expected rate of return. All model inputs are forward-looking: the risk-free rate, the equity premium, and the market beta of the asset.

You need to memorize the CAPM formula.

The CAPM formula tells you what investors care about: comovement with the market.

The CAPM specifically ignores the stand-alone risk (standard deviation) of your project. That is, investors do not care about your projects' variance, because they are smart enough to diversify away any idiosyncratic risk. Investors care only about your project's market betas, because it is betas that measure the component of risk that your project contributes and that investors holding the wide market portfolio would not have diversified away.

Mechanically, it looks sensible.

On a pragmatic level, the CAPM is seductive. It limits your attention to just two benchmark assets. It gives you a coherent universal measure of where projects lie on the spectrum between stocks and bonds. More market beta means "more like stocks," and thus higher expected rates of return ("just like stocks"). Less market beta means "more like bonds," and thus lower expected rates of return ("just like bonds").

There are deeper CAPM rationales that economists call "equilibrium models."

Without going into detail, economists also love a deep "economic equilibrium model" justification for the CAPM that I will largely spare you. In this view, financial markets are perfect, each and every investor faces the same tradeoffs and uses the model, and each and every asset is priced by it. When all the assumptions are satisfied, it implies mathematically that the CAPM must hold. Necessarily, there could then not be any benchmarks other than the risk-free rate and the stock market, and the only valid measure of risk would be the market beta. This CAPM justification, with its stringent assumptions, is too orthodox and simply not realistic.

The CAPM is a "legacy" model.

Unfortunately, more important than philosophy, the empirical data soundly rejects the CAPM, as I will explain below in more detail. For now, let me just say that you must still study the CAPM not only because it is conceptually important but also because every finance dinosaur in the real world is using it — and, more than likely, it will growl questions about the CAPM in your job interview.

☺author = dinosaur, too.

Q 10.1. What are the assumptions underlying the CAPM? Are the perfect market assumptions among them? Are there more?

The Security Market Line (SML)

A first quick use of the CAPM formula.

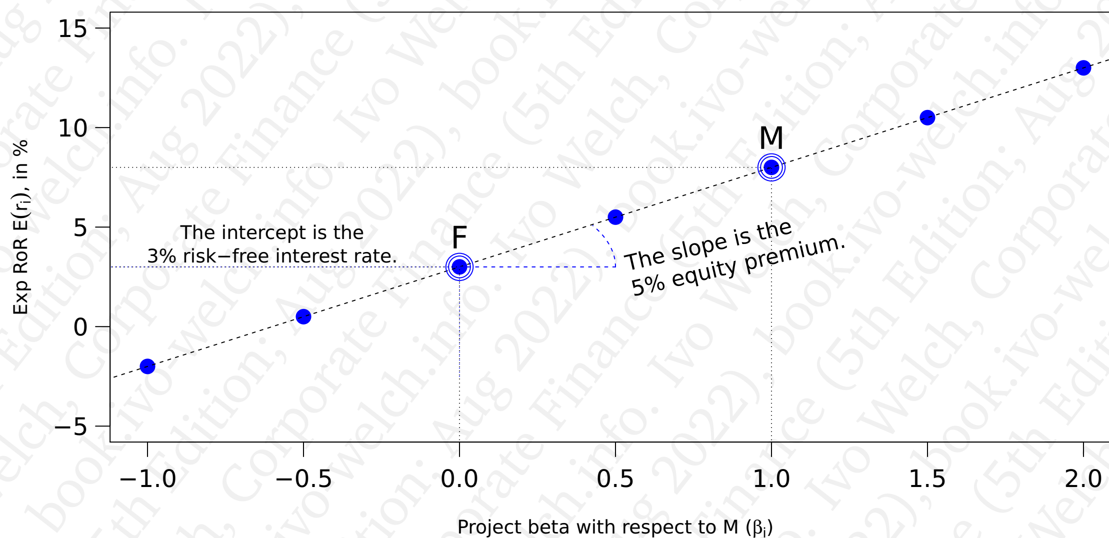
Let's first use the CAPM formula as a recipe. If you believe that the risk-free rate is 3% and the expected rate of return on the market is 8%, then the CAPM states that

$$\mathsf{E}(r_i) = 3\% + (8\% - 3\%) \cdot \beta_i = 3\% + 5\% \cdot \beta_i$$
$$\mathsf{E}(r_i) = r_F + [\mathsf{E}(r_M) - r_F] \cdot \beta_i$$

Therefore, a project with a beta of 0.5 should have a cost of capital of $3\% + 5\% \cdot 0.5 = 5.5\%$, and a project with a beta of 2.0 should have a cost of capital of $3\% + 5\% \cdot 2.0 = 13\%$. The CAPM gives the opportunity cost for your investors' capital: If the project with the beta of 2.0 cannot earn this expected rate of return of 13%, you should not take this project and instead return the money to your investors. Your project would add too much risk for its reward. Your investors have better opportunities elsewhere.

The SML is the CAPM.

The CAPM formula is often graphed as the **security market line** (**SML**), which shows the relationship between the expected rate of return of a project and its beta. Figure 10.1 draws a model-perfect security market line for seven assets. Each investment asset (such as a stock or a project) is a point in this coordinate system. Because all assets in our example properly follow the CAPM formula, they must lie



		Investment Asset						
		A	B	F	C	M	D	E
Market Beta	β_i	–1.0	-0.5	0.0	0.5	1.0	1.5	2.0
Expected Rate of Return	$E(r_i)$	–2.0%	0.5%	3.0%	5.5%	8.0%	10.5%	13.0%

Figure 10.1: ***The Security Market Line With Perfect Knowledge.*** This graph plots the CAPM relation $E(r_i) = r_F + [E(r_M) - r_F] \cdot \beta_i = 3\% + (8\% - 3\%) \cdot \beta_i$, where β_i is the beta of an individual asset with respect to the market. In this graph, we assume that the risk-free rate is 3% and the equity premium is 5%. Each point is one asset (such as a stock, a project, or a mutual fund). The point M could be the value-weighted market portfolio or any other security with a $\beta_i = 1$. F could be the risk-free asset or any other security with a $\beta_i = 0$.

on a straight line. The SML is just the graphical representation of the CAPM formula. The slope of this line (the reward per unit of beta) is the equity premium, $E(r_M) - r_F$, and the intercept is the risk-free rate, r_F.

If you knew the true expected quantities, and the CAPM held, the SML would be a straight line. The data to estimate it is a scatterplot.

Alas, in the real world, even if the CAPM holds, you would not have the data to draw Figure 10.1. The reason is that you do not know true expected returns and true expected market betas. Figure 10.2 plots a version where you have to rely only on what most investors have and rely on — observable historical data averages. Thus you can only fit an “estimated security market line,” not the “true security market line.” And you have to hope that your historical data has provided good, unbiased estimates of the true forward-looking market beta and true forward-looking expected rates of return. (Both are big assumptions!) If the fitted line on the data looks straight, you would not immediately throw out the CAPM. In any case, any workable version of the CAPM in real life can only state that there should roughly be a linear relationship

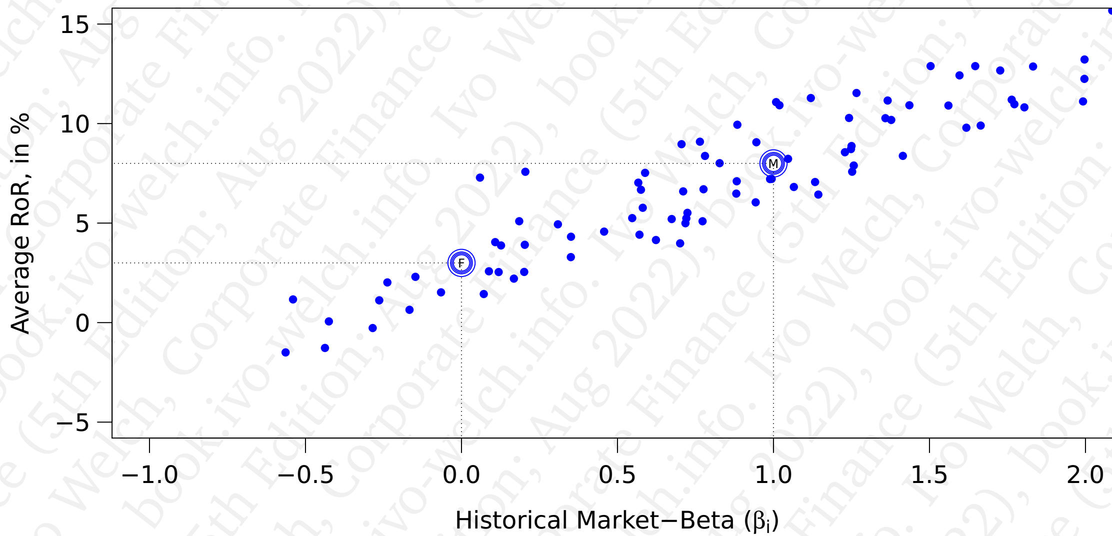


Figure 10.2: ***Data To Estimate A Security Market Line in an Ideal CAPM World.*** Assuming the CAPM indeed held, then this plot shows what you would be, at best, confronted with: All you could observe would be historical average returns and historical betas. Even for past (ex-post) expected returns and past (ex-post) betas, the realizations are just noisy measurements. It's worse for (ex-ante) forward-looking expected returns and expected market-betas. You then have to hope that past average returns and past average betas are also representative of future expected returns and future betas. There is always hope...

between the data-estimated market betas and the data-estimated expected rates of return, just as drawn in Figure 10.2.

Q 10.2. Assume the CAPM holds. The risk-free rate is 4%. What is the appropriate cost of capital for a project that has a beta of 3 if

1. The expected rate of return on the market is 7%.
2. The expected rate of return on the market is 12%.
3. The equity premium is 3%.

Q 10.3. The risk-free rate is 4%. The expected rate of return on the market is 12%. What is the cost of capital for a project that has a beta of –3? Does this make economic sense?

Q 10.4. If you plot historical excess rates of return against historical market-betas, do you get a perfectly straight line?

Q 10.5. The risk-free rate is 4%. The expected rate of return on the market is 7%. A corporation intends to issue publicly traded bonds that *promise* a rate of return of 6% and offer an *expected* rate of return of 5%. What is the implicit beta of the bonds?

Q 10.6. Draw the SML if the risk-free rate is 5% and the equity premium is 9%.

Q 10.7. What is the equity premium, both mathematically and intuitively?

The CAPM in the Present Value Formula

If you take the CAPM at face value, it gives you a good denominator for the NPV formula, the opportunity cost of capital, $E(r)$:

We usually use the CAPM output, the expected rate of return, as our discount rate.

$$NPV = C_{t=0} + \frac{E(C_{t=1})}{1 + E(r_{t=1})} + \frac{E(C_{t=2})}{1 + E(r_{t=2})} + \cdots$$

(Subscripts here are time rather than firms.) Together, the CAPM and the NPV formulas tell you that cash flows that correlate more with the overall market are of less value to your investors and therefore require higher expected rates of return ($E(r)$) in order to pass muster (well, to pass the hurdle rate, which is determined by the alternative opportunities that your model presumes your investors have).

➤ Asset-Pricing Model, Pg.238.

The CAPM is called an **asset-pricing model**, even though it is most often expressed in terms of a required expected rate of return rather than in terms of an appropriate asset price. Fortunately, the two are equivalent — you can always work with the CAPM return first, and then discount the expected cash flow into an appropriate price. A given expected rate of return implies a given price. (If you do not know the fair price, you will have to take two aspirin and work with a more difficult version of the CAPM formula. It is called **CEV** and is explained in the companion online chapter.)

It is easier to work in required returns than in prices.

☺or something more hallucinogenic

Equity and Asset Betas

As in Section 9.5, it is important that you always distinguish between asset costs of capital and equity costs of capital. And, fortunately, just as you can take weighted averages of expected rates of return, you can take weighted averages of betas. Thus, whatever worked in Section 9.5 with the overall costs of capital also works here with market betas. Done. You can skip the rest of this section, or endure a few more examples.

➤ Asset and equity betas, Formula 8.5, Pg.210.

For example, assume that the risk-free rate is 4% and the equity premium is 5%. You own a \$100 million project with an asset beta of 2.0 that you can finance with \$20 million of risk-free debt. Truly risk-free debt always has a beta of 0. To find your equity beta, write down the formula for your asset beta (firm beta):

Don't use the equity beta to estimate your project's hurdle rate. Use the asset beta instead.

$$\underbrace{\beta_{Firm}}_{2.0} = \underbrace{\left(\frac{\text{Debt value}}{\text{Firm value}}\right)}_{20\%} \cdot \underbrace{\beta_{Debt}}_{(0)} + \underbrace{\left(\frac{\text{Equity value}}{\text{Firm value}}\right)}_{80\%} \cdot \underbrace{\beta_{Equity}}_{(\beta_{Equity})}$$

Solve this to find that your market beta of equity is 2.5. It is this market beta of equity that you would find reported on sites like *YAHOO!* FINANCE. You would not want to base your hurdle rate for your entire firm's typical average project on your equity beta: Such a mistake would recommend you use a hurdle rate of $E(r_i) = r_F + [E(r_M) - r_F] \cdot \beta_i = 4\% + 5\% \cdot 2.5 = 16.5\%$. This would be too high. Instead, you should require your average projects to return $E(r_i) = 4\% + 5\% \cdot 2.0 = 14\%$.

➤ Typical, average, and marginal betas, § 13.4, Pg.372.

	20% Debt	80% Equity	100% Project
Beta	0.0	2.5	2.0
⇒ Cost of Capital	4%	16.5%	14.0%

In both cases, the capitalization-weighted average of debt and equity rate of return is always the overall project asset rate of return.

If you use comparables, first unlever them.

Conversely, if your project is private but the potential future owners are well-diversified, you may have to find its hurdle rate by looking at public comparables. Let's presume you find a similarly sized firm with a similar business that *YAHOO!*FINANCE lists with a beta of 2, or perhaps better yet, the firm's industry. Remember that financial websites always list only the equity beta. The CAPM tells you that the expected rate of return on the equity is $4\% + 5\% \cdot 2 = 14\%$.

Finance some with lower-cost debt and some with higher-cost equity.

However, even in a perfect capital market, this is not necessarily the hurdle rate for your project. When you look further on *YAHOO!*FINANCE, you may notice that your comparable is itself also financed with 90% debt and 10% equity. (Companies in financial distress and banks often have leverage that is this unusually high. If the comparable had very little debt, then a debt beta of 0 might have been a good assumption, but unfortunately, in this case it is not.) Corporate debt rarely has good historical return data that would allow you to estimate a debt beta. Consequently, practitioners often estimate the expected rate of return on debt via debt comparables based on the credit rating. Say your comparable's debt is rated BB and say that BB bonds have offered *expected* rates of return of 100 basis points above the Treasury. (This might be 200 basis points *quoted* above the Treasury). With the Treasury standing at 4%, you would estimate the comparable's cost of capital on debt to be 5%. The rest is easy. The expected rate of return on your project should be

➤ Credit ratings, § 6.2, Pg.123.

$$E(r_{Project}) = 90\% \cdot 5\% + 10\% \cdot 1.4\% = 5.9\%$$
$$= w_{Debt} \cdot E(r_{Debt}) + w_{Equity} \cdot E(r_{Equity})$$

This would make a good hurdle rate estimate for your project.

➤ Typical, average, and marginal betas, § 13.4, Pg.372.

Does Risk Reduction Create Value?

Diversification reduces risk, but does not create value.

In the 1960s and 1970s, many firms became **conglomerates**, that is, companies with widely diversified and often unrelated holdings. Can firms add value through such diversification? The answer is "usually no." Diversification indeed reduces the standard deviation of the company's rate of return (diversified companies are less risky). Yet, in a perfect market, your investors can just as well diversify risk for themselves. They don't need the firm to do it for them. This is a more general perfect-market insight than the details of what follows. Again: if investors can do it without the firm, the firm cannot add value by doing it for them.

Belabor the obvious?

As in the previous section, we can elaborate on this in the context of the CAPM. However, the basic idea should hold in any reasonable framework, e.g., if projects have different cash flow horizons and thus different term-based costs of capital. Thus, you can consider it "done" and you can skip this section, too, if you already fully understand this. Otherwise, endure the example below.

If your $900 million firm ABC with a beta of 2 and a risk of 20% is planning to take over the $100 million firm DEF (e.g., with a beta of 1 and also a risk of 20%), the resulting firm is worth $1 billion. ABC + DEF indeed has an idiosyncratic risk lower than 20% if the two firms are not perfectly correlated, but your investors (or a mutual fund) could just have held 90% of their portfolios in ABC and 10% in DEF and thereby achieved the very same diversification benefits. If anything, a merger takes away your investors' freedom: They no longer have the ability to buy, say, 50% of their portfolios in ABC and 50% in DEF. (In a perfect CAPM world, this would not matter, because any investor could simply repurchase the firm and undo this.) The CAPM makes it explicit that the cost of capital does not change unduly. Say both firms follow the CAPM pricing formula, and say that the risk-free rate is 3% and the equity premium is 5%,

A specific diversification example worked out for you, in which projects are priced fairly, and diversification neither creates nor destroys value.

$$E(r_{ABC}) = 3\% + 5\% \cdot 2 = 13\%$$
$$E(r_{ABC}) = r_F + [E(r_M) - r_F] \cdot \beta_{ABC}$$

and

$$E(r_{DEF}) = 3\% + 5\% \cdot 1 = 8\%$$
$$E(r_{DEF}) = r_F + [E(r_M) - r_F] \cdot \beta_{DEF}$$

The newly formed company will have an expected rate of return (cost of capital) of

$$E(r_{ABC\,+\,DEF}) = 90\% \cdot 13\% + 10\% \cdot 8\% = 12.5\%$$
$$E(r_{ABC\,+\,DEF}) = w_{ABC} \cdot E(r_{ABC}) + w_{DEF} \cdot E(r_{DEF})$$

➤ Value-weighted portfolios, § 8.5, Pg.209.

(This is not a property of the CAPM but a property of expected rates of return. It would also work with the simpler benchmarking approach of the previous chapter.)

For the combined conglomerate, the market beta is now

Duh.

$$\beta_{ABC\,+\,DEF} = 90\% \cdot 2 + 10\% \cdot 1 = 1.9$$
$$\beta_{ABC\,+\,DEF} = w_{ABC} \cdot \beta_{ABC} + w_{DEF} \cdot \beta_{DEF}$$

The merged company will still follow the CAPM,

$$E(r_{ABC\,+\,DEF}) = 3\% + 5\% \cdot 1.9 = 12.5\%$$
$$E(r_{ABC\,+\,DEF}) = r_F + [E(r_M) - r_F] \cdot \beta_{ABC\,+\,DEF}$$

Its cost of capital has not unduly increased or declined. In an ideal [CAPM] world, no value has been added or destroyed — even though ABC + DEF will have a risk lower than the 20% per annum that its two constituents had.

(Diversification could add value if risk-averse investors are too foolish to diversify themselves — a possibility which the CAPM has assumed away. With foolish investors, the conglomeration could make these investors' portfolios more diversified and less risky, making them better off. However, although some such an investor may exist, it seems implausible that they are plentiful and important enough to determine pricing in the world.)

Deconstructing Quoted Rates of Return

As in Section 9.6, the CAPM provides just an underpinning to the expected rate of return, not the quoted rate of return. (In fact, the example I constructed in Section 9.6 was surreptitiously based off the CAPM. I even told you the market-beta in the text as an aside. With a market-beta of 0.13, the expected rate of return on the risky bond was $0.13 \cdot 12\% \approx 1.58\%$.)

Short-Term and Long-Term Projects?

Coercing the CAPM to be more practical.

Although the CAPM formally recognizes only one SML in theory, we use different risk-free rates for different project horizons in practice. Thus, short-term projects would have lower costs of capital than long-term projects. For example, you might assess, say, a 3% equity premium and use prevailing rates for the Treasuries — say a one-year Treasury at 1% and a thirty-year Treasury of 2% — and thus assess

		Your Project β_i					
		–2	–1	0	1	2	3
Short-Term Projects	$E(r_i) = 1\% + \beta \times 3\%$	–5%	–2%	1%	4%	7%	10%
Long-Term Projects	$E(r_i) = 2\% + \beta \times 3\%$	–4%	–1%	2%	5%	8%	11%

Recall that we are not sure whether we should use the same equity premium (here 3%) for both near and far project cash flows. It is due to ignorance that we typically use the same equity-premium estimate regardless of an equity premium that depends on the term.

Q 10.8. A corporate bond with a beta of 0.2 will pay off next year with 99% probability. The risk-free rate is 3% per annum, and the equity premium is 5% per annum.

1. What is the price of this bond?
2. What is its promised rate of return?
3. Break down the bond's quoted rate of return into its components.

Q 10.9. Suppose that going to your school has total (including opportunity) costs of $30,000 *this year and upfront*. With 90% probability, you are likely to graduate from your school. If you do not graduate, you have lost the entire sum. Graduating from the school will increase your 40-year lifetime annual salary by roughly $5,000 per year, but more so when the market rate of return is high than when it is low. For argument's sake, assume that your extra-income beta is 1.5. Assume the risk-free rate is 3%, and the equity premium is 5%. What is the financial value of your education?

Nerdnote: If the CAPM truly held, long-term bonds would have higher expected rates of return than short-term bonds, and this could be explained exactly by their positive market beta. Alas, researchers and text-book writers do not know why long-term bonds have had negative market betas for a few decades now. However, be aware that applying the CAPM to long-term bonds would so obviously contradict reality that few bond investors have been tempted to use it in this context. Instead, bond investors heavily use adjusted yield-curve estimates.

10.3 Estimating the Extra Input: Market Beta

We already discussed estimating the risk-free rate and equity premium in the previous chapter. We also discussed beta estimation in the chapter before. Nevertheless, because beta is the only novel aspect relative to benchmarking, let's discuss it a little more. Beta tells you how the rate of return of your project fluctuates with that of the overall market. Unlike the previous two inputs, which are the same for every project in the economy, the beta input depends on your own specific project characteristics: Different projects have different betas.

Unlike the risk-free rate and the equity premium, beta is specific to each project.

Just as with the risk-free rate and the expected rate of return on the stock-market (or equivalently, the equity premium) in Chapter 9, investors are really interested in the *future* market betas of your projects and not in their historical market betas. No one really cares about the past for its own sake. But as usual, you often have no choice other than to rely on estimates, and these are usually based largely on statistical analysis of historical data.

The CAPM has three inputs. We will cover them in detail.

➤ Will history repeat itself?, § 7.6, Pg.167.

First, the good news. Betas can be forecast much more accurately than average rates of return (which we use as estimates of expected rates of return). Use daily rates of return and shrink them towards 1, and you are good, especially for more diversified portfolios. (Or look them up at https://www.ivo-welch.info/research/betas/.) Now the bad news. The further ahead you need to estimate, the less reliable betas become. See, betas drift themselves slowly over horizons of many years. It's like shooting at a moving target — not as good as shooting at a fixed target, but if the target is moving slowly enough, it shouldn't end up too bad. *C'est la vie.*

Betas are reliable over short periods.

Market Beta Estimation Based on Historical Data

The basic mechanics of finding the historical market beta for a project with historical rates of return is easy. You run a **market-model** regression. The independent variable is the rate of return on the stock market. The dependent variable is the rate of return on your project. It is also good practice to subtract the risk-free T-bill rate from both your project's and the stock market's rates of return. Any statistical software package (and common computer spreadsheet programs like Excel or Openoffice) can readily calculate the coefficients a and b in the market-model regression:

Ways to estimate beta.

$$\underbrace{(r_{\text{Project}} - r_F)}_{\text{y variable}} = a + b \cdot \underbrace{(r_{\text{Market}} - r_F)}_{\text{x variable}}$$

The slope b is the market beta. It's a good thing that we use b as a symbol instead of β, because the b that the regression spits out is only an estimate of a true beta (β), and not the true and unknowable beta itself.

This is only the basics. To get a better forward-looking market beta estimate, you should do the following:

Historical market beta to forward market beta

1. Use daily stock returns, not monthly stock returns.
2. Use about two years' worth of data. Between one and five years of data will do.
3. For memory sake, I earlier recommended a "shrink" of 50% towards 1.0. A more exact shrink of your first-pass market beta would be 30-40%, depending on the timing of the cash flow that you intend to use it on:

Corporate Finance (5th Edition), book.ivo-welch.info, Ivo Welch, Aug 2022

< 1 Year	$(1-0.3)\times b + 0.3\times 1$
> 5 Years	$(1-0.4)\times b + 0.4\times 1$

This 0.3 (or 0.4) factor is used partly because it reduces the impact of historical outliers, and partly because true market betas drift over long horizons. If you want, you can shrink beta by another 10% if your project and firm are small (to compensate for additional estimation "noise").

For example, if your statistical software gives you a first-pass market-beta estimate for your project of 2.0, and you want to estimate a CAPM cost of capital for a project cash flow in 1 year, then use $(1-0.3)\times 2.0 + 0.3 = 1.7$. If you want to estimate it for a project cash flow in 10 years, use 1.6. If your first-pass estimate is -1.0, and the cash flow is in 1 year, use $(1-0.3)\times -1.0 + 0.3 = -0.4$.

It does not matter much which particular stock-market index you use as your independent variable. The S&P 500 with or without dividends is fine. There are also other more sophisticated methods, but the above three guidelines cover the most important basics. It is unlikely that you can improve much on them. These market betas are as good as simple estimates are going to get.

What not to do!

In practice, you may encounter two common estimation practices that dramatically worsen the quality of estimated market betas. So let me warn you:

1. If you have good daily data, do not estimate market beta with monthly return data. (And if you have no choice [as, for example, with hedge funds, which report rates of return only monthly], then shrink more — think 50-60%, not 30-40%.)
2. If you have your firm's own stock returns, do not use industry portfolio returns as stand-ins for your firm. Although industry betas move less than stock-specific betas and thus seem appealing, in reality industry betas (or peer betas) are much worse predictors for stocks than the stocks' own market betas.

If you see either practice, tell the dinosaur using them that the mammals are taking over and they'd better evolve and adapt!

Market Beta Estimation Based on Theoretical Consideration

Turn the formula around to help contemplate beta

As a corporate manager, you are rarely interested in the market beta of an industry or even a stock. Usually, you are interested in the market beta of a potential project. Sometimes, your firm may not even be publicly traded, so you would not have any historical price data to begin with. In this case, corporate CAPM users sometimes rely on economic intuition rather than historical statistics. To see the logic, rearrange the CAPM formula. Now, do you think your project cash flows and its future project values (which are influenced by changes in the economy) are likely to move more or less with the overall stock market (and possibly the overall economy)?

$$\mathsf{E}(r_{\text{Project}}) = r_F + [\mathsf{E}(r_M) - r_F]\cdot\beta_{\text{project}} \iff \beta_{\text{Project}} = \frac{\mathsf{E}(r_{\text{Project}}) - r_F}{\mathsf{E}(r_M) - r_F}$$

The right side of this formula helps translate your intuition into a beta estimate. What rate of return (above the risk-free rate) will your project have if the market were to

Nerdnote: If you want to estimate future market-beta even better, then shrink not towards 1, but towards a smaller constant, like 0.6-0.8, if your firm and project are small.

have +10% or –10% rate of return (above the risk-free rate)? Clearly, such guesswork is difficult and error-prone — but it can occasionally provide a market-beta estimate when no other is available. But be aware that such estimates are almost always poor.

Don't be so confident!

If you do not believe me that your estimate is going to be so bad that you may as well just go back to the peer benchmarks from Chapter 9, then I dare you to try. Randomly pick five stocks from *YAHOO!*FINANCE. Do not peek at their market betas. Explain to me what they should be, and then check your claims against their actual market betas. If you can accurately assess which market betas are farther from 1.0, then you are a better intuitive economist than I am. In fact, I have almost no economic intuition as to why entire asset classes, such as long-term bonds, have had negative market betas over the last 20 years and positive market betas before then.

Do you even want beta for a project without historical stock returns?

Moreover, *please* stand back and think for a moment what you are really doing here. If you are dealing with a new project that has never seen the light of day and that has no historical data, would you even want to use the CAPM? And are you a fully diversified unconflicted owner-investor who cares only about market-risk and not about idiosyncratic project risk, and who has access to a perfectly competitive capital market? There are whole sections of the economy that cannot meet these criteria. In particular, if you are an entrepreneur who satisfies them, I would really like to meet you — I have never met such an entrepreneur. (Besides, are you already so convinced by the beauty and logic of the CAPM that you are willing to believe that it is a good description of the real world — even though I have shown you zero empirical evidence about the model itself up to this point?)

Q 10.10. According to the CAPM formula, a zero-beta asset should have the same expected rate of return as the risk-free rate. Can a zero-beta asset still have a positive standard deviation? Does it make sense that such a risky asset would not offer a higher rate of return than a risk-free asset in a world in which investors are risk-averse?

Q 10.11.A comparable firm (with similar size and business) has a *YAHOO!*FINANCE-listed equity beta of 2.5 and a debt/asset ratio of 2/3. Assume that the debt is risk-free.

1. Estimate the equity beta for your firm if your projects have similar betas, but your firm will carry a debt/asset ratio of 1/3.
2. If the risk-free rate is 3% and the equity premium is 2%, then what should you use as your firm's hurdle rate?
3. What should investors demand as the expected rate of return on the comparable firm's equity and on your own equity?

Q 10.12. You own a stock portfolio that has a market beta of 2.4, but you are getting married to someone who has a portfolio with a market beta of 0.4. You are three times as wealthy as your future significant other. What will be the beta of your joint portfolio?

10.4 Neutralizing Equity-Premium Uncertainty

The three most important numbers.

Do you recall my claim that the risk-free rate and the equity premium were the two most important numbers in finance, regardless of whether you are using the CAPM or not? Well, you also want to know the market-beta for the same reason. It is an extremely useful number, too.

Beta has another interesting use

➤ Shorting, § C, Pg.176.

➤ Exchange-Traded Fund, § 7.1, Pg.151.

It is very easy to short the stock market (e.g., using an S&P 500 future or ETF). This allows you to "innoculate" or "hedge" your project against overall stock-market risk. Just short the right amount of stock, which is exactly the ratio that market beta gives you. For example, if you have $100 million invested in an asset with a market beta of 3, you can short $3 \cdot \$100 = \$300$ million in the market and thereby reduce your market risk to zero. If the stock market happens to go down by 1%, you would expect (a) your project to go down by 3% but (b) your hedge to go up by the same $3 \cdot 1\%$. The CAPM formula even suggests that your equity-premium estimate is now irrelevant.

...but it may increase idiosyncratic risk

However, a short market position can also increase the variance of your project outcomes: You may end up in a scenario in which your own project underperforms and the stock market outperforms. You may even go bankrupt because of it. Your project's idiosyncratic-risk component and your errors in estimating betas now become more important. This is not a problem if your project owners are highly diversified, and your particular project is just a tiny fraction of their wealth that they don't care a great deal about. Yet, it is a problem if they are not; or if you, as the corporate manager, care about your one specific project a great deal (or if there are bankruptcy costs, as you will learn in Chapter 19).

You can even neutralize the CAPM market-risk!

Putting this together, from your perspective as the CEO of one small company in a large market, you can render a degenerate version of the CAPM formula to be nearly true by definition. If you are shorting the correct amount of stock market, it won't matter whether you are overestimating or underestimating the equity premium. The limits to this strategy are your estimation uncertainty about beta and your idiosyncratic risk tolerance. In the real world, a full short may neither be possible nor desirable. If you do not immunize your company against market risk, then it matters to you what the equity-risk premium is — and whether the CAPM is right in the first place.

But should you be short the market?

You may object that you would not want to short the stock market — betting against the market was historically not a smart maneuver. But, as a CFO, do you really know better whether you should be long or short the stock market? If it is fairly priced, so be it. Leave this choice to your investors. If they want to bet on or against the overall stock market, they do not need you to do it for them. You are only "abusing" the insights of market-betas to avoid or at least reduce the consequences of your ignorance about your project's and the overall market's cost of capital.

Nerdnote: The strategy of neutralizing the market could work only for a small number of companies and investors. If it became too widespread, it would change the investment opportunity set.

10.5 Is the CAPM the Right Model?

The CAPM Assumptions Are Not Innocuous

Everyone can only invest in the same things.

Although the CAPM edifice is reasonable, it does not mean that this edifice "obviously" holds. The CAPM model leans a lot more on the perfect-market assumptions (and then some) than our earlier chapters did. Are most financial markets really so perfect? Do most investors really hold diversified stock-market portfolios? Do they really care *only* about risk and reward in their financial-asset portfolios and nothing else?

Using particular stocks as insurance for you.

Stand back for a moment. How can the CAPM edifice crumble? Consider the following examples:

- If you own a house, chances are that much of your current wealth is invested in the equity portion of your house, and you are not as diversified as you should be. You should then try to find stocks that reduce your house risk exposure, not stocks that reduce your financial market risk exposure. You should like stocks that go up when your house value goes down.
- If you are under 40 years of age, chances are that much of your lifetime wealth is in your human capital. It is not diversified. And only *you* can invest easily in *your* education: No one else can. *You* need to hedge *your* career, not mine. *I* need to hedge *my* career, not your's. You should like stocks that go up when the value of your expertise goes down.
- If you are a tech engineer and work in Silicon Valley, you should short technology stocks as a hedge against their tanking. Conversely, you should not mind losing in your financial portfolio when technology stocks boom (and you end up rich from your employer's stock options, anyway). Yet many engineers in Silicon Valley are so irrationally overconfident, excited, and/or convinced of technology and their (stock-picking) abilities that they end up buying mostly technology stocks for their portfolios, instead. They double up rather than hedge. It may have worked for them so far, but just wait…
- Do firms really live in near-perfect capital markets? Entrepreneurs often need to scrape together whatever capital they can. If they cannot easily find many capital providers, they may have to pay much higher costs of capital than suggested by the CAPM. And they may be forced to invest most of their own wealth — to the point of bankrupting themselves if their projects fail.
- Entrepreneurs are notorious for staking their entire life's savings on their startups. They are hardly ever diversified and usually short of funds.

So, even though the theoretical CAPM assumptions are simple enough to be appealing, their applicability is actually quite narrow — the CAPM considers a scenario in which all investors do not care about anything but the risk and return in the public financial markets, and they all have (largely) the same investments and investment opportunities. Don't think the CAPM *has* to be true just because it seems reasonable.

Beta, schmeta.

What if every investor chose portfolios for personal reasons and not with the same perspective — some looking to hedge their houses, others their job, others their industry, others their product's failure? Then it may well be that some assets offer higher or lower expected rates of return than suggested by the CAPM — the CAPM would not hold. In this case, corporate managers really should not rely on the CAPM. Instead, they should stick to a more holistic approach (e.g., higher hurdle rates) or the less ambitious peer benchmarking approach from Chapter 9.

The Empirical Evidence Firmly Rejects the CAPM Among Stocks

No more bedtime stories. Here comes the truth.

Sadly (for finance professors), it indeed turns out to be true that the empirical evidence does not support the CAPM. Thus the best advice in real life is not to use the CAPM. It does not work. Use the benchmarking approach instead.

Huh? WTH? Did you really read me right?

Yes you did. Don't use the CAPM. The empirical evidence firmly contradicts it. This means that the common corporate use of the CAPM — to obtain hurdle rates for projects in the NPV formula — is based on no more than wishful thinking. It is contrary to empirical evidence.

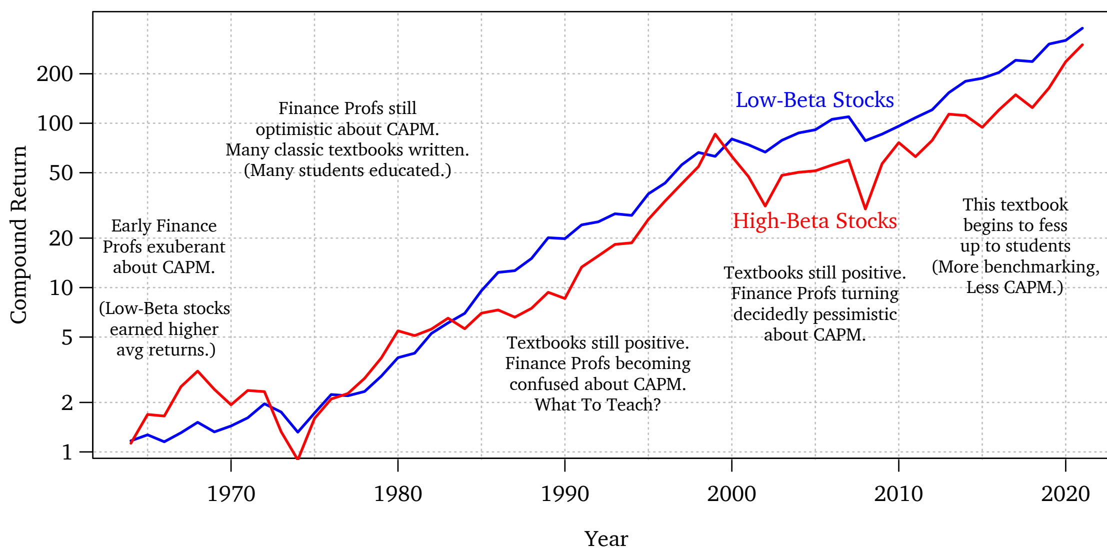


Figure 10.3: ***Portfolio Performance of High-Beta and Low-Beta Stocks, 1964–2021.*** This graph plots the compound performance of portfolios formed from the lowest-quintile beta stocks and the highest-quintile beta stocks. High-beta stocks were indeed more exposed to stock price movements. If the CAPM had been right, higher-beta stocks should also have offered higher expected performance. However, they did not. The graph even understates the evidence — the average bullish stock market in this sample period was positive. Even without higher risk compensation, high-beta stocks should have outperformed low-beta stocks because the stock market did well. (I shudder at how much worse high-beta stocks would have performed in a bear market!) **Source:** Kenneth R. French Data Library.

It does not take a PhD to see this (though it takes many to hide and explain it away). You won't find this kind of figure in other textbooks that want to obscure the truth.

I could summarize decades worth of academic work, but understanding the principal problem does not require a higher degree. It only requires basic common sense and open eyes. At its most basic implication, the CAPM says that high-beta stocks should offer higher average rates of return than low-beta stocks.

To show that this was not so, I downloaded the market-beta portfolios (publicly posted by the famous researcher Ken French). Ken sorts stocks by market-betas into quintile portfolios (here value-weighted) and then tracks the return performance of these portfolios over the following year. Figure 10.3 plots them. The realized ex-post market-beta was 0.75 for the low-beta quintile and 1.47 for the high-beta quintile. Market-betas were stable. Because stocks delivered almost 12% per annum from 1964 to 2021, high-beta stocks should have delivered about $(1.47-0.75) \cdot 12\% \approx 10\%$ more per annum than low-beta stocks. Instead, they delivered *less*.

The simplest test.

This is a robust finding. Within the asset class of stocks, the empirical evidence shows that higher market-beta stocks did not have higher average rates of return in the past than lower market-beta stocks. It's not just the more subtle CAPM problem that benchmarks other than the equity market *also* matter; it's the very unsubtle problem that beta itself does not seem to matter. Yikes 😬! Yikes 😬! Yikes 😬!

The problem is not subtle or sensitive to how researchers analyze the data.

Ergo, using a market-beta of 1 on every project (the ultimate shrinkage) would not have done any harm predicting future average returns. Standing in the past, if you had wished to predict future expected rates of return, you would have done better assuming that every stock had a market-beta of 1. This would have been the same as benchmarking all stocks indiscriminately to the VFIAX (S&P 500) index. Standing here today in 2022, I think that this is still good advice about future average stock returns.

Please just assume a beta of 1 == benchmark

Important

Although one can predict the market-beta of individual stocks quite well, you should use a good estimate of market-beta only for diversification and hedging purposes (neutralizing the equity premium).

For estimating differential expected rates of return (costs of capital) within the asset class of publicly-traded stocks, you should skip the beta estimation and use your estimate of the expected rate of return on the market instead. Another way of saying this is to use the CAPM formula with the (obviously incorrect) input as if every market beta input was 1.0.

➤ In Defense of the Use of the CAPM

If the evidence is against the CAPM, then why do we finance professors torture you with it? We may indeed have sadistic streaks (as our captive PhD students can testify), but this is not why. This "why" is much easier to answer than how stocks are priced in the real world or what the best estimate of the appropriate hurdle rates for your project should be.

😊Sadists!

***Across* asset classes:** The CAPM was not rejected across asset classes. Stocks had higher average rates of return than bonds which had higher average rates of return than bills. In this sense, some high-beta assets offered higher average returns than some low-beta assets.

Caveat: Although this is evidence that investors have indeed been rewarded for taking on more beta-risk, almost any measure of risk would have also predicted this to be the case. Declaring victory for the CAPM based on this rough evidence is overreaching.

Impeccable intuition: The CAPM shines through its simplicity and focus on diversification. It gets executives away from the false notion that public investors care about the idiosyncratic risk of projects that they can diversify away. Thus, corporate diversification into a conglomerate for its own sake can reduce its own risk but not market risk. It cannot add value. Investors can diversify themselves. They don't need the firm to do it for them. The CAPM logic reinforces this important point.

Evidence-based or faith-based?

Strong Belief: Many instructors and practitioners find the CAPM to be so plausible that they are willing to live with the "absence of CAPM evidence." They do not take this absence to mean "evidence of CAPM absence." Thus, they adopt the CAPM based on faith and not on evidence — actually, more like contrary to the evidence about the basic association between market-betas and average rates of return shown above. If you do this, you must be aware that this is what you are doing.

A crutch

Stand-in for Expected Cash Flow Default: The CAPM often assigns higher costs of capital to projects that are more likely to fail. If you have not fully adjusted your expected cash flow estimates downwards to account for the possibility of for failure (a common human as well as managerial error), the CAPM cost of capital may help imposing a higher hurdle rate on riskier cash flows. It's a crutch.

Another crutch

Stand-in for Imperfect-Market Factors: The CAPM often assigns higher costs of capital to projects that do not satisfy the perfect-market assumptions and that face higher costs of capital. Again, this can accidentally result in better cost-of-capital estimates not *because* of the CAPM, but *despite* the CAPM. It's another crutch.

Avoid Duplication

Such a Great Idea: The CAPM is so intuitive and appealing that it would be "rediscovered" again and again by those who were not forced to learn it. Those who cannot remember the past are condemned to repeat it

Everyone uses it: The CAPM is *the* standard. Exhibit 10.4 shows that over 80% of the CFOs of large companies report that they always or almost always use the CAPM. Multifactor models are sort of like the CAPM, but throw in a few more factors beyond the market factor (e.g., changes in oil prices). The problem is that every firm may have its own factors, so multifactor models can give very different results for different users. Beyond the CAPM, no model is in wide use (and this includes specific versions of its multifactor cousin known as the **APT**). Consequently, you have no choice but to understand the CAPM model well — *if you will work for a corporation, then the CAPM is the benchmark model that your future employer will likely use and will expect you to understand well*. Chances are that you will be interrogated about it in your job interview.

☺Dinosaurs of the world — unite!

Again, the CAPM is simply *the* standard. The CAPM is also used as a benchmark by many investors (e.g., to rate their investment managers), government regulatory commissions, courts (in tort cases), and so on. It is literally the dominant, if not the only, widely used model to estimate the cost of capital. There is even a whole section on the CFA exam about the CAPM!

Alternatives — please stand up: The famous social psychologist Kurt Lewin wrote that "there is nothing more practical than a good theory." If not the CAPM, then

Method	Usage Frequency by CFOs: Large Firms (Sales > $1 b)	Smaller Firms
CAPM (Ch. 10)	(83%)	(27%)
Multifactor Model (Companion)	(69%)	(29%)
Historical Average Returns (Ch. 9)	(33%)	(17%)
Investor Expectations	(30%)	(18%)
Dividend Discount Model (Ch. 4)	(18%)	(13%)
Market Return	(17%)	(21%)
Regulatory	(14%)	(19%)
CAPM (Mukhlynina and Nyborg, CFR 2016)	(87%)	

Figure 10.4: ***CFO Valuation Techniques for the Cost of Capital.*** Not reported, use of the CAPM is more common among managers with an MBA — and in firms who rely on consultants who in turn use the CAPM. **Source:** John Graham, 2021, except last line

what else? There are alternatives, but none are universally accepted. (My own recommendation is to go with the benchmarking approach from the previous chapter or to use the [incorrect] market-beta of 1.0 in the CAPM formula for every stock. It's not a great theory, but it has predicted better.)

Market Hedging: Even if the market beta does not measure the average rate of return, it does guide managers about how much market risk they face — and, if they so desire, how to neutralize it and focus on their real expertise.

Be aware that my treatment of the CAPM in an introductory corporate finance textbook borders on heresy. Most textbooks still make the CAPM their centerpiece. They do this not because the authors believe in it, but because it is dogma that new finance students (and many old finance professors) are too fragile to deserve the hard truth. I am sorry — I wish I could have told you a happy bedtime story about how the world is nice and orderly, too. But it would have been a lie.

Do you want a bedtime story that "the world is ok" in order to be able to go to sleep, or the tough truth?

➤ If you must use it...

If you still want to use the CAPM, here is my advice. As a corporate executive, you should always first think hard about why and when you want to use the CAPM. Think about whether it is useful for your own cost-of-capital estimates. Think about whether the CAPM errors seem too large to be useful for your particular needs. And understand what you are getting. Do simpler benchmarks first — do they agree with the CAPM estimate?

Never make the following errors, please.

Accuracy: The CAPM is a poor model if you want precision. If you believe that CAPM expected rates of return should be taken seriously on any digits after the decimal point, then you need a shrink. Please realize that, at best, the CAPM could only ever offer expected rates of return that are in the ballpark, plus or minus a few percentage points, perhaps. And maybe not even that...

Don't expect accuracy and don't use it for financial investing.

"Cost of Capital" Expert Witnessing

When Congress tried to force the "Baby Bells" (the split-up parts of the original AT&T) to open up their local telephone lines to competition in 1996, it decreed that the Baby Bells were entitled to a fair return on their infrastructure investment — with fair return to be measured by the CAPM. (The CAPM is either the *de facto* or legislated standard for measuring the cost of capital in many other regulated industries, too.) The estimated value of the telecommunication infrastructure in the United States is about $10 to $15 billion. A difference in the estimated equity premium of 1% may sound small, but even in as small an industry as local telecommunications, it meant about $100 to $150 million a year — enough to hire hordes of lawyers and valuation consultants opining in court on the appropriate equity premium. Some of my colleagues bought nice houses with the legal fees.

I did not get the call. I lack the ability to keep a straight face while stating that "the equity premium is exactly x point y percent," which was an important qualification for being such an expert. In an unrelated case in which I did testify, the opposing expert witness even explicitly criticized my statement that my cost-of-capital estimate was an imprecise range — unlike me, he could provide an exact estimate for the equity premium, and it was 11.0% per year! (His clients were happy!) *Baby-Bell History: Bradford Cornell, UCLA*

☺"Outrun not the bear, but you" joke

Actually, if accuracy is important, you are in trouble no matter what. Finance does not have *any* models that can offer physics-level precision for stocks. Fortunately, you may not have to be good at estimating value; you may just need to be better than your competitors. Always remember that valuation of risky long-term projects is as much an art as it is a science. And you wouldn't be the first investor or corporate executive who just happened to be saved by Lady Luck, even if the bet was not a particularly good one.

Avoid using the CAPM for short-term financial investment purposes.

Investment purposes: If you are not a corporate executive looking to determine your project hurdle rate, but a financial investor looking for good investments from the universe of financial instruments, and with an ability to shift your money around every day, then please do not use the CAPM. Although the CAPM offers the correct intuition that wide diversification needs to be an important part of *any* good investment strategy, there *are* better investment strategies than just investing in the overall market index. These are discussed in advanced investments courses.

➤ Mean-variance optimization in detail, § 8.2, Pg.192.

Please do not confuse the CAPM with the mean-variance framework discussed in Chapter 8, although the latter is one of the CAPM's foundations. Mean-variance optimization is an asset-selection technique for your individual portfolio, and it works, regardless of whether or not the CAPM holds.

➤ Corporate Time-Varying Costs of Capital, § 5.4, Pg.102.

Longer-Term Differences: If you are a corporate executive, be especially cautious about discount rates for expected cash flows far in the future. Look at your cost of capital more holistically. And even if you use the CAPM for guidance, remember that the CAPM has two terms.

The first term is the risk-free rate, which applies to all projects, regardless of beta. Fortunately, this one is easy. You should use higher costs of capital for cash flows that will occur in the more distant future. And you have a great estimate of the premium that long-term projects need to offer over short-term projects, based on the Treasury yield curve. You don't even need historical estimates: you can use the prevailing Treasury yield curve. *Use it! It works!*

It is the second term (the beta multiplied by the risk-premium) — i.e., your beta risk-adjustment — that you must be especially suspicious about. If your cash flows will occur in many years, be modest. Do not over-rely on the risk assessment from the CAPM. Cut down extreme estimates. Shrink and shrink again (towards the average rate of return on risky investments). (Of course, do not forget to be similarly humble in your expected cash flow estimates.) Fortunately, you may be ok:

- As a corporate manager, compare the cost of capital on *your equity* vs. the cost of capital on *your debt* for your long-term cash flows. With an equity premium based on the performance of stocks vs. long-term Treasuries of about 1-2% from 1970 to today, it may not matter so much whether your project A has a beta of 0.8 and your project B has a beta of 1.2. The implied cost-of-capital difference between these two projects of under $(1.2-0.8)\cdot 2\% \approx 1\%$/year is already small and probably swamped by your expected cash flow estimation error.
- For long-term cash flows — occurring in, say, 10-20 years — your best estimate of your equity market betas should be tilted even further towards 1.0. If you estimate historical OLS market beta to be 0.5 for A and 1.5 for B today, you may well want to use a market beta (a) shrunk to around 0.7 and 1.3 for predicting future market betas; and (b) shrunk to around 0.9 for A and 1.1 for B for use in the CAPM expected-return formula. Think about this: A and B would now have a difference in the implied cost of equity capital of $0.2\cdot 2\% \approx 0.4\%$. This is way below your noise-and-uncertainty threshold.

  But let's continue. Say your projects are partly debt-financed, too. Now you need to calculate asset market betas rather than equity market betas. Let's say both projects have 50% debt that is almost risk-free. Then your asset beta would be $0.5\cdot0.0+0.5\cdot0.9 = 0.45$ for A and $0.5\cdot0.0+0.5\cdot1.1 = 0.55$ for B. Now you have a project cost-of-capital difference $(0.55-0.45)\cdot 2\% \approx 0.2\%$ between A and B.

Asset betas for high-beta stocks are much lower — and they often give CAPM estimates some (sorely needed) time-stability.

How does this expected rate-of-return difference between A and B compare to your own uncertainty about your projects' relative expected cash flows? Does the CAPM beta risk-adjustment really matter much in light of your uncertainty?

The estimated CAPM cost of capital for long-term cash flows is fragile.

In sum, cash flows in the more distant future and cash flows that have higher market-betas should likely be discounted more, as already explained in Chapter 9. But be humble about your capabilities in trying to distinguish between projects that are similar along time and asset-class dimensions.

Taking Advantage of CAPM Violations

The CAPM leans heavily on equilibrium market forces that are really quite weak.

Can investors earn free money (through what is called **arbitrage**) if the CAPM fails? Not really. The universe remains intact even if the CAPM does not hold, and even in a perfect market.

Q: What happens if a stock offers too much or too little expected rate of return? A: Investor stampedes towards or away from the stock.

What would happen in the CAPM if one stock offered more than its appropriate expected rate of return? Its price would be too low. It would be too good a deal. Investors would immediately flock to it, and because there would not be enough of this stock in the economy, investors would bid up its price. This would lower its expected rate of return. The price of the stock would settle at the correct CAPM expected rate of return. Conversely, what would happen if one stock offered less than its due expected rate of return? Investors would not be willing to hold enough of this stock: The stock's price would be too high, and its price would fall. Neither situation should happen in a CAPM world.

Assets not priced according to the CAPM do not allow you to make money for nothing. However, they could imply good deals.

Is this a free-money arbitrage? No! When stocks do not follow the CAPM formula, buying them remains risky. Yes, some stocks would offer a higher or lower expected rate of return and thus seem to be too good or too bad a deal, attracting too many or too few investors. (Or, the investors may not even flock towards better deals, perhaps because they have other needs, perhaps because they are asleep at the switch.) But these "CAPM-too-cheap" (or "too expensive") stocks would remain risky bets, and smart risk-averse individual investors would want to buy just a little more (or less) of them. They could not earn risk-free profits. There would be no arbitrage here. The market forces working on correcting any CAPM mispricing are just modest.

Maybe for some?!

And also remember that there are good reasons why the CAPM would not hold in the first place. For example, as we have discussed, it relies heavily on many perfect-market assumptions. If investors are taxed or liquidity-constrained (that is, they cannot easily diversify, e.g., because they are invested in a startup or family firm) or do not agree on the inputs, then it is quite plausible that some firms or even sectors (such as "value-type firms" or "growth-type firms") could offer higher or lower expected rates of return than the CAPM suggests. But not all is lost. It may mean that if *you* are an investor with CAPM preferences, you can do a little better than holding the overall market portfolio by tilting your market-like portfolio just a little towards stocks that offer higher expected rates of return than suggested by the CAPM formula and just a little away from stocks that offer lower expected rates of return.

➤ value and growth stocks, § 7.3, Pg.159.

10.6 Is Beta Useless Without the CAPM?

Do not confound market-beta usefulness with CAPM usefulness.

Importantly, market-beta is not useless if the CAPM does not hold. After all, it was never supposed to be a measure of reward. It is a measure of risk contribution for someone holding the stock market. If you are an investor who holds something close to the market portfolio, market-beta gives you the asset's risk-contribution noxiousness. If you use it to tilt your portfolio towards low-beta stocks, you are creating a portfolio that offers similar overall expected rates of return but at a lower overall portfolio standard deviation. It is also useful for "immunizing" your portfolio against market-wide stock returns.

Just don't believe the CAPM E(r)

If the CAPM does not hold, what fails is the market-beta's usefulness in a corporate context for measuring the cost of capital. That is, the manager can no longer use it

to infer the preferences and behavior of her investors, and therefore can no longer use it to infer what expected rate of return they demand. *C'est la vie*. What to do then? My advice is to fall back on what you learned in the previous chapter. Try to find the closest benchmarks you can instead of fixating on the stock market.

10.7 CAPM Alternatives and Perspectives

You have already learned in the previous chapter about the principal alternative to the CAPM — benchmarking. The CAPM is really based on similar ideas, but it pushed the idea too far. It was too bold and overconfident.

Madness: One step beyond. Can models be overconfident?

CAPM vs. Benchmarking: Widening and Narrowing Concepts

The CAPM both generalizes and narrows the idea of benchmarking. The generalization is that market beta is a more universal and objective measure of how equity-like any investment asset is than subjective judgment. It should work for any asset — be it bond, stock, one specific stock or fund, equity options, gold, art, etc. The narrowing is that the CAPM is very specific about the fact that it is market beta — and market beta alone — that is the benchmark of the risk that investors care about. No other factors or exposure to other factors matter.

Stocks do seem to give higher average returns than bonds, but betas do not seem to predict higher expected returns among stocks.

If the CAPM model is correct, then using more benchmark portfolios (*à la* Chapter 9) than just the stock market would still be just fine. Each benchmark portfolio would be priced according to the CAPM and lie on the SML. It is merely a convenience of the CAPM that you do not *have to* worry about these benchmark portfolios. If you do use these other benchmarks, fine. If you do not, fine, too. You will still obtain the exact same proper expected rate of return.

If the CAPM model is incorrect, then by using it, you would have gone one step too far. You could easily get the wrong answer. For example, say, investors do not care about market risk (and market beta), but only about oil risk, computer technology risk, and biotech risk. It could be the case that because the market portfolio contained some of these risks, it provided a higher expected rate of return. But it would really matter now whether your project and market beta come from oil risk (which may give a higher expected rate of return) or, say, gold risk (which may not). The CAPM would give you the right answer only if your project happened to have the same proportions as the market portfolio in its exposures.

What you really need for value comparisons are the benchmark portfolios that matter most and fit best. Of course, unlike the CAPM, the benchmark portfolio method can also be harder to use: What is the best benchmark asset? But benchmarking would still work in principle — just as long as you give this method all the right benchmark portfolios as input!

In a sense, the CAPM does only one thing. It tells you what the correct benchmark is — market beta — based on the very common-sense notion that investors should like stocks that are less exposed to market risk. If you use the CAPM, you are assuming it is correct; and, if it is, your estimates become easier and possibly even a little more accurate. But if the model is not correct, you have bet on the wrong horse and you may end up working with an incorrect cost of capital.

My Personal Opinion about Costs of Capital

Caveat Emptor!

Now I will give you my own educated opinion about good project cost-of-capital estimates. Beware: Different finance professors may come to different conclusions, so do not take my opinion as the gospel.

➤ Solid Inference — Little Disagreement

What is solid empirical evidence?

The following expected-return premia are rock-solid and not primarily a matter of opinion:

- There definitely is a time value of money.
- There definitely is a term structure. Long-term cash flows usually require higher costs of capital than short-term cash flows. Your investors can earn higher expected rates of return elsewhere for longer-term commitments.
- There definitely is a credit component. Assets have to make up for higher probabilities of default with higher promised yields — that is, higher yields when they succeed.

➤ Market Imperfections, § 11, Pg.275.

We have not covered the following yet. It will be explained in Chapter 11.

- Market imperfections play important roles. There are many kinds. Here are a few examples. There seems to be a liquidity premium. Assets that can be quickly liquidated (especially in general market crashes like 1987 or 2008) are more expensive, and different asset classes seem to have different degrees of liquidity. Because of their collateral, mortgage bonds tend to have lower costs of capital than general bonds. Firms with less access to capital markets, such as startups, seem to pay higher costs of capital, although adjusting for default makes this difficult to measure. Investors pay more in personal income tax for interest receipts than they do for capital gains, which makes equities relatively more desirable and reduces their after-tax income. Sentiment and agency considerations also seem to play important roles in equity trading. Many of these market imperfections embody some concepts of risk, but it is not the market beta. Interestingly, courts agree with imperfect-market views. They allow as much as a 20-30% discount for the value of privately held assets relative to publicly traded peers. We may not know what the costs of capital for small, privately held firms are, but we do know that they are usually much higher.

➤ Uncertain Inference — More Disagreement — My Subjective Assessments

The equity premium seems too hard to predict.

I wish I knew the equity risk premium — and for a lot of different reasons. The CAPM is only one of them. Benchmarking is another. Alas, I am not so confident that I have a good assessment. We are dealing with finance (with estimated probabilities), not physics (with known probabilities). And expected rates of return, such as those on the stock market, are notoriously unreliable.

Market beta times equity premium is probably small, after other premia.

After taking into account the premia just mentioned (which includes premia that are sometimes included in and have to be captured by the risk premium, but which I already have in my number), the remaining risk premium — especially over longer horizons — is probably relatively small (1-2%). However, we do not know for sure. Our uncertainty is much larger than our certainty about its magnitude. And you

need to realize that betas for cash flows far into the future are much closer to 1 than historical regressions would suggest. The "CAPM" beta-metric for measuring the project's risk impact and expected rate of return is only of modest (if any) importance.

Use reasonable risk adjustments — a little bit of beta, a little bit of idiosyncratic risk, a little bit more heuristic finesse.

So what would I do (unless forced to use the CAPM by a superior)? My best alternative cost-of-capital recommendation would start out just like the CAPM: As the first term in a formula, I would recommend that you use the rate of return on bonds of similar maturity as the cash flow that you want to value. Usually, this means that you assign higher costs of capital to cash flows farther in the future. It is only with the second term — the equity risk-adjustment — that I would tinker. Instead of the (shrunk) CAPM market beta multiplied by some historical equity premium (of 1-3%/year geometric above long-term Treasuries), I would recommend a more holistic approach.

- Take into consideration that projects with high volatility and/or with high leverage are more risky. The equity of these projects probably requires a higher expected rate of return to keep your investors happy. Realize that projects with higher idiosyncratic risk are also usually the same projects about which executives tend to be most overly optimistic. (Check again: are you sure your expected cash flows in the NPV numerator are not biased by your overconfidence?)
- As a manager, take into consideration whether you and your owner-shareholders are well-diversified. If the owners have most of their wealth in your firm or project, then you should require higher rates of return for riskier projects—but in this case, it is not "beta risk" that matters, but "total risk (standard deviation)."
- Take into consideration that public investors may "like" certain types of investments — like crypto or **environmental, social, governances** (**ESG**) related investments — and are often willing to pay higher prices and thus accept lower average rates of return for some such projects. If they are willing to give you money at lower expected rates of return, take this into consideration.

There is probably little harm if you calculate a (repeatedly shrunk) CAPM market beta and apply it to a relatively low equity premium (say, 2%/year) for some heuristic orientation. Assess whether any other non-CAPM cost-of-capital assessments seem reasonably similar to such a CAPM assessment. In this sense, the CAPM can still be informative.

➤ If Forced

What would I do?

- If I ran a large publicly-traded firm with good access to capital markets and I needed to assess the cost of capital for a typical medium-term project, I would assume an equity premium of 1-3% per annum and apply the resulting expected rate of return on the stock market (i.e., adding back the fixed-income borrowing rate) to the equity components of all my long-term cash flows. The exception would be projects for which I would have strong prior knowledge that they are very unlike stocks.

  I would consider long-term corporate debt to have a higher cost of capital than equivalent Treasuries, but a lower cost of capital than my own equity — the

book.ivo-welch.info, Ivo Welch, Corporate Finance (5th Edition, Aug 2022)

latter primarily because debt provides a corporate income tax shield (as you will learn in Chapter 18) and not because the equity premium over long-term corporate bonds is high.

➤ Income Taxes and Cost of Capital, § 18, Pg.553.

And I would distinguish between the cost of capital and the hurdle rate (which one should not do a perfect capital market). We will discuss this further in the next three chapters.

- If I ran a startup firm funded by myself or other-not-so-rich investors (so that the startup would risk a large fraction of personal wealths), I would never even think about the CAPM. Risk would definitely play a central role, but not in the market-beta and comovement sense. I would be more concerned about total risk and funding uncertainty.

  I would perhaps assume a cost of capital of 2% to 6% above the *expected* rate of return on my uncollateralized debt, depending on the project risk and my risk aversion. I would probably require an even higher hurdle rate — it could easily be in the double digits. Even more radically, I might even abandon NPV-based models altogether and worry more about the downside of losing my shirt.

  My perspective would change again when I was about to sell the firm to better-diversified investors (such as a venture capitalists). I would try to figure out not just my own but also their perspective. They often base their valuations on alternative projects in the same industry and do a lot of benchmarking and comparables, the subjects of Chapters 9 and 15. However, it is here that I would begin relying more on traditional NPV modeling, too.

➤ Comparables, § 15, Pg.451.

And I would never use any of my schemes here (or the CAPM) for the pricing of bonds, derivatives, real-estate, human capital, or other non-stock-like kinds of projects.

NPV or Comparables? Nobel Prize laureate Eugene Fama thinks comparables are better.

Am I the only professor who recommends against using the CAPM? No. Many do so in conversations among themselves and with other experts, and even more do when their own money is on the line. However, many have bee concerned to admit to our *collective* ignorance in front of students. It is easier to teach the intuition and beauty of the CAPM and stop than it is to dismiss it right after teaching it. (And teach it, we must!) Let me appeal to a higher authority for backup: Eugene Fama — the most famous finance professor alive, winner of a Nobel prize, and partly responsible for the original spread of the CAPM — nowadays strongly recommends against the CAPM, too. His view is that using the CAPM expected rate of return as your cost of capital in an NPV calculation effectively divides one bad uncertain number by another bad uncertain number. This practice convolutes errors and uncertainty about expected cash flows in the numerator with errors and uncertainty about expected returns in the denominator. If you get lucky, your errors cancel. If not, they do not. Yikes 😬! "Gene" prefers comparables.

Conclusion

Important

- The CAPM is *the* benchmark model in the real world. Most corporations use it.
- Interviewers will likely expect you (a student applying for a job) to understand the CAPM. Regardless of whether the model holds or not, you have to know it.
- The empirical evidence suggests that the CAPM is not a good model for predicting expected rates of return.
- The first CAPM term (the time adjustment) seems to hold better than the second CAPM term (the risk adjustment).
- Market betas tend to revert back towards 1 over time. This requires you to shrink ordinary OLS beta estimates very aggressively towards 1.
- The geometric equity premium above long-term Treasuries (for evaluating long-term cash flows) has been — and is unlikely to be more in the future than — 2-3% per annum.
- The CAPM never offers great accuracy. Do not lean on or trust the CAPM.
- Mean-variance optimization (Section 8.2) works even if the CAPM does not. If you are an investor who dislikes risk, the immediate implication is that instead of the typical market portfolio, you should tilt towards stocks with lower betas and away from stocks with higher betas. This will give you an overall portfolio that has lower risk for the same amount of expected rate of return.
- Peer portfolio benchmarking (Chapter 9) works regardless of whether the CAPM does or does not work.
- You may or may not want to immunize your project against equity-premium risk and estimation uncertainty, using its beta estimate. Immunized projects have much clearer cost-of-capital benchmarks than unimmunized projects.

Q 10.13. Does the empirical evidence suggest that the CAPM is correct?

Q 10.14. If the CAPM is wrong, why do you need to learn it?

Q 10.15. Is the CAPM likely to be more accurate for a project where the beta is very high (say, above 1), one where it is very low (say, below 0), or one where it is zero?

Q 10.16. To value an ordinarily risky project, that is, a project with a beta in the vicinity of about 1, what is the relative contribution of your personal uncertainty (lack of knowledge) about (a) the risk-free rate, (b) the equity premium, (c) the beta, and (d) the expected cash flows? Consider both long-term and short-term investments. Where are the trouble spots?

Ivo Welch, Corporate Finance (5th Edition), book.ivo-welch.info, Aug 2022

Summary

This chapter covered the following major points:

- The CAPM provides an "opportunity cost of capital" for investors, which corporations can use as the cost of capital in the NPV formula. The CAPM formula is

  $$E(r_i) = r_F + [E(r_M) - r_F] \cdot \beta_i$$

  Thus, there are three inputs: the risk-free rate of return (r_F), the expected rate of return on the market ($E(r_M)$), and the project's or firm's market beta (β_i). Only the latter is project-specific.

- The line plotting expected rates of return against market beta is called the security market line (SML).

- The CAPM provides an expected rate of return, consisting of the term premium and the risk premium. It ignores the default premium. In the NPV formula, the default risk and default premium work through the expected cash flow in the numerator, not through the expected rate of return (cost of capital) in the denominator.

- For r_F, you should use bonds that match the timing of your project's cash flows. Thus, cash flows farther in the future often require higher opportunity costs of capital. Even if you do not believe in the CAPM, term adjustment is important.

- The expected rate of return on the stock market is a critical CAPM input if the project's market beta is high — but the best estimate of the equity premium itself remains elusive. There are many guesstimation methods, but no one really knows which one is best. Reasonable estimates for the equity premium ($E(r_M) - r_F$) can range from about 1% to 8% per annum, although 2-3% seems most common for cash flows more than a few years into the future.

- There are a number of methods to estimate market beta. Don't be too confident about betas far from 1, especially for long-term project cash flows.

- If you combine a short position in the stock market with a positive-beta project, the combined project is a lot easier to price than a project with a positive beta. By effectively creating a combined project with a zero market-beta, you can neutralize the effects of "CAPM is wrong model" and equity-premium errors.

- Never believe the CAPM blindly. Its estimates are poor. Use them more for "general direction" than as "accurate guides." Think compass, not GPS.

- Even though its estimates are poor, understand the CAPM well. Everyone will expect you to.

This negative perspective on the CAPM is so uncommon in a textbook (but not among the experts actually studying the models) that it is important that you don't misunderstand what this chapter says. So let's end this chapter with a FAQ:

- **Q**: Should riskier cash flows not require higher promised rates of return?

  > **A**: Riskier projects have to promise higher rates of return, i.e., offer higher default premiums. This is not the same as higher risk premiums in the CAPM sense. In NPV applications, make sure to reflect the default risk in the expected cash flow numerator. Riskier projects need to pay off a lot more when they succeed, just to make up for the fact that they fail more often.

- **Q**: Should long-term and therefore riskier cash flows not require higher expected rates of return?

  > **A**: Long-term projects command term premiums. Thus, in NPV applications, you should usually use higher required costs of capital for more distant cash flows. You should not use the CAPM for this. The U.S. Treasury Yield Curve

gives you a working first estimate about how much extra premium long-term cash flows should require above short-term cash flows.

- **Q**: Besides a term premium (and perhaps a leverage-structure adjustment to account for debt capacity), should riskier stocks and corporate cash flows have higher expected discount rates?

  A: Maybe, but be careful. First, remain humble and make your estimate err on the side of modesty. Don't be overconfident in your ability to judge equity risks. If you can judge the risks well, make sure your estimates first flow into your expected cash flows in the NPV numerator. Second, don't be too wedded to the CAPM for the extra "risk-premium kicker." Instead, combine your cost-of-capital estimate with judgment, perhaps also looking at factors such as volatility (especially if your owners are not fully diversified). If I am vague here, it is deliberate. I cannot give you clear universal guidance.

- **Q**: If CAPM cost-of-capital estimates are so bad, why has this not hurt companies that rely on the CAPM more badly?

  A: Most companies use a hurdle rate considerably higher than their cost of capital estimates. That is, even though they pay lip service to the CAPM, they don't really use it as their end-all.

Preview of the Chapter Appendix in the Companion

The appendix to this chapter explains:

- How the "certainty equivalence value" (CEV) allows you to use the CAPM for projects that you are not buying at the appropriate equilibrium price. For example, you would need the CEV to work out how to value an inheritance that will be higher if the economy does well. (Just because the inheritance is "free" to *you* does not mean that there is a zero value to it.)
- How to use the CEV formula to estimate the value of a project for which you have historical cash flows, but no market value information.
- How the CAPM is derived from the fact that the optimal portfolio is always the combination of two portfolios, one of which may be the risk-free asset.
- What a few more CAPM alternatives are and how to use them. The first alternative is the APT (arbitrage pricing theory) and its relative, the Intertemporal CAPM. The second alternative is a "Fama-French"-style model, which uses factors such as value, growth, momentum, investment, and robustness. This Fama-French model seems to predict better than any alternatives, but it is less grounded in theory (or, you may say, reason) than the former. It also often gives counterintuitive results — e.g., that small growth stocks are safer than large value stocks and therefore that managers should use *lower* discount rates on, say, risky tech ventures.

Keywords

apt, p.18; arbitrage, p.22; asset-pricing model, p.7; capital asset pricing model, p.3; capm, p.3; certainty equivalence, p.7; conglomerate, p.8; environmental, social, governance, p.25; esg, p.25; market-model, p.11; security market line, p.4; sml, p.4.

Answers

AQ 10.1 Yes, the perfect market is an assumption underlying the CAPM. In addition,

1. Investors are rational utility maximizers.
2. Investors care only about overall portfolio mean rate of return and risk at one given point in time.
3. All parameters are known (not discussed until later in the chapter).
4. All assets are traded. Every investor can buy every asset.

AQ 10.2

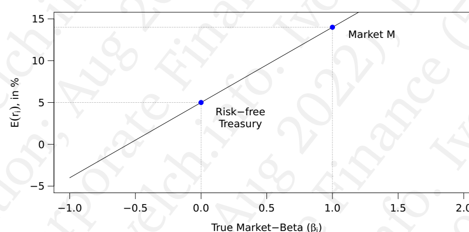
1. With $r_F = 4\%$ and $E(r_M) = 7\%$, the cost of capital for a project with a beta of 3 is $E(r) = r_F + [E(r_M) - r_F] \cdot \beta_i = 4\% + (7\% - 4\%) \cdot 3 = 13\%$.
2. With $r_F = 4\%$ and $E(r_M) = 12\%$, the cost of capital for a project with a beta of 3 is $E(r) = r_F + [E(r_M) - r_F] \cdot \beta_i = 4\% + (12\% - 4\%) \cdot 3 = 28\%$.
3. Same answer as (a).

AQ 10.3 With $r_F = 4\%$ and $E(r_M) = 12\%$, the cost of capital for a project with a beta of –3 is $E(r) = r_F + [E(r_M) - r_F] \cdot \beta_i = 4\% + (12\% - 4\%) \cdot (-3) = -20\%$. Yes, it does make sense that a project can offer a negative expected rate of return. Such projects can be good investments that you would even be willing to expect losses on average, because of the great insurance that they are offering against the drops in the value of your (market-type) portfolio.

AQ 10.4 No — this plot would be about real-world historical data and not about (true) expectations. It would be a scatterplot. If the CAPM holds, a straight, upward-sloping line should fit the scatterplot points well.

AQ 10.5 Write down the CAPM formula and solve $E(r_i) = r_F + [E(r_M) - r_F] \cdot \beta_i = 4\% + (7\% - 4\%) \cdot \beta_i = 5\%$. Therefore, $\beta_i = 1/3$. Note that we are ignoring the promised rate of return.

AQ 10.6 The security market line is



AQ 10.7 The equity premium, $E(r_M) - r_F$, is the premium that you can expect to earn on the risky and long-term overall stock market portfolio above and beyond what you can expect to earn on no-risk Treasuries.

AQ 10.8 It does not matter what you choose as the per-unit payoff of the bond. If you choose \$100, you expect it to return \$99.

1. Thus, the price of the bond is $PV = \$99/(1 + [3\% + 5\% \cdot 0.2]) \approx \$95.19$.
2. Therefore, the promised rate of return on the bond is $\$100/\$95.19 - 1 \approx 5.05\%$.
3. The risk-free rate is 3%, so this is the term premium (which contains any inflation premium). The (expected) risk premium is 1%. The remaining 1.05% is the default premium.

AQ 10.9 The cost needs to be discounted with the current interest rate. Because payment is upfront, this cost is \$30,000 now! The appropriate expected rate of return for cash flows (of your earnings) is $3\% + 5\% \cdot 1.5 = 10.5\%$. You can now use the annuity formula to determine the PV if you graduate:

$$\frac{\$5{,}000}{10.5\%} \cdot \left[1 - \left(\frac{1}{1 + 10.5\%}\right)^{40}\right] \approx \$47{,}619 \cdot 98.2\% \approx \$46{,}741.46$$

With 90% probability, you will do so, which means that the appropriate risk-adjusted and discounted cash flow is about $42,067.32. The NPV of your education is therefore about $42,067.32 – $30,000 = $12,067.32.

AQ 10.10 Yes, a zero-beta asset can still have its own idiosyncratic risk. And, yes, it is perfectly kosher for a zero-beta asset to offer the same expected rate of return as the risk-free asset. The reason is that investors can hold gazillions of assets, so the idiosyncratic risk of the zero-beta asset will just diversify away.

AQ 10.11 This is an asset beta versus equity beta question. Because the debt is almost risk-free, we can use $\beta_{\text{Debt}} \approx 0$.

1. First, compute an unlevered asset beta for your comparable with its debt-to-asset ratio of 2 to 3. This is $\beta_{\text{Asset}} = w_{\text{Debt}} \cdot \beta_{\text{Debt}} + w_{\text{Equity}} \cdot \beta_{\text{Equity}} = (2/3) \cdot 0 + (1/3) \cdot 2.5 \approx 0.833$. Next, assume that your project has the same asset beta, but a smaller debt-to-asset ratio of 1 to 3, and compute your own equity beta: $\beta_{\text{Asset}} = w_{\text{Debt}} \cdot \beta_{\text{Debt}} + w_{\text{Equity}} \cdot \beta_{\text{Equity}} \Rightarrow 0.833 \approx (1/3) \cdot 0 + (2/3) \cdot \beta_{\text{Equity}} \Rightarrow \beta_{\text{Equity}} = 1.25$.
2. With an asset beta of 0.83, your firm's asset hurdle rate should be $E(r_i) = 3\% + 2\% \cdot 0.83 \approx 4.7\%$.
3. Your comparable's equity expected rate of return would be $E(r_{\text{Comps Equity}}) = 3\% + 2\% \cdot 2.5 = 8\%$. Your own equity's expected rate of return would be $E(r_{\text{Your Equity}}) = 3\% + 2\% \cdot 1.25 = 5.5\%$

AQ 10.12 Your combined matrimonial beta would be $\beta_{\text{Combined}} = (3/4) \cdot 2.4 + (1/4) \cdot 0.4 = 1.9$.

AQ 10.13 No, the empirical evidence suggests that the CAPM does not hold. The most important violation is that, on average, high-beta stocks did not earn higher rates of return than low-beta stocks. (Worse — they even failed to earn higher average rates of return in a stock market that outperformed Treasuries by 3-6% per year on average!)

AQ 10.14 Even though the CAPM is empirically rejected, it remains the benchmark model that almost everyone uses in the real world. Moreover, even if you do not trust the CAPM itself, at the very least it suggests that covariance with the market could be an important factor.

AQ 10.15 The CAPM should work very well if beta is about 0. The reason is that you do not even need to guess the equity premium if this is so.

AQ 10.16 For short-term investments, the expected cash flows are most critical to estimate well (see Section 4.1 on Page 59). In this case, the trouble spot (d) is really all that matters. For long-term projects, the cost of capital becomes relatively more important to get right, too. The market betas and risk-free rates are usually relatively low maintenance (though not trouble-free), having only modest degrees of uncertainty. The equity premium will be the most important problem in the cost-of-capital estimation. Thus, the trouble spots for long-term projects are (b) and (d).

End of Chapter Problems

Q 10.17. If the CAPM holds, then what should you do as a manager if you cannot find projects that meet the hurdle rate suggested by the CAPM?

Q 10.18. In a perfect world (and in the absence of externalities, which would imply that projects influence other projects), should you take only the projects with the highest NPV?

Q 10.19. Write down the CAPM formula. Which are economy-wide inputs, and which are project-specific inputs?

Q 10.20. The risk-free rate is 6%. The expected rate of return on the stock market is 8%. What is the appropriate cost of capital for a project that has a beta of 2?

Q 10.21. The risk-free rate is 6%. The expected rate of return on the stock market is 10%. What is the appropriate cost of capital for a project that has a beta of –2? Does this make economic sense?

Q 10.22. Draw the SML if the true expected rate of return on the market is 6% per annum and the risk-free rate is 2% per annum. What would the figure look like if you were not sure about the expected rate of return on the market?

Q 10.23. A junk bond with a beta of 0.4 will default with 20% probability. If it does, investors receive only 60% of what is due to them. The risk-free rate is 3% per annum and the risk premium is 5% per annum. What is the price of this bond, its promised rate of return, and its expected rate of return?

Q 10.24. What would it take for a bond to have a larger risk premium than default premium?

Q 10.25. A corporate zero-bond promises $1,000 in one year. Its market beta is 0.3. The equity premium is 4%; the equivalent Treasury rate is 3%. According to the CAPM, what is the appropriate bond price today?

Q 10.26. Explain the basic schools of thought when it comes to equity premium estimation.

Q 10.27. If you do not want to estimate the equity premium, what are your alternatives to finding a cost-of-capital estimate?

Q 10.28. Explain in 200 words or less: What are reasonable guesstimates for the market risk premium and why?

Q 10.29. Should you use the same risk-free rate of return both as the CAPM formula intercept and in the equity premium calculation, or should you assume an equity premium that is independent of investment horizon?

Q 10.30. Should a negative-beta asset offer a higher or a lower expected rate of return than the risk-free asset? Does this make sense?

Q 10.31. An unlevered firm has an asset market beta of 1.5. The risk-free rate is 3%. The equity premium is 4%.

1. What is the firm's cost of capital?
2. The firm refinances itself. It repurchases half of its stock with debt that it issues. Assume that this debt is risk-free. What is the equity beta of the levered firm?
3. According to the CAPM, what rate of return does the firm have to offer to its *creditors*?
4. According to the CAPM, what rate of return does the firm have to offer to its *levered equity holders*?
5. Has the firm's weighted average cost of capital changed?

Q 10.32. Does the empirical evidence suggest that the CAPM is correct?

Q 10.33. Why do you need to understand the CAPM?

Q 10.34. Download daily stock-market data for Tesla and the S&P 500 for calendar years 2020 and 2021 from *YAHOO!* FINANCE.

1. What was Tesla's plain stock-market-model regression beta in your sample in each year?
2. What was Tesla's shrunk stock-market beta? Use a shrinkage factor of 0.5 towards a market beta of 1.0 and your just-calculated estimate.
3. Was the shrunk market-beta in 2020 a better predictor of the market-beta in 2021 than the plain market-beta?
4. If Tesla had a debt-equity ratio of 1-to-2 and its debt was close to risk-free, what was its asset beta? (Hint: To determine the debt-to-asset ratio, make up an example in which a firm has, say, a 30% D/E ratio.)

Q 10.35. A peer firm in a comparable business has an equity beta of 2.5 and a debt-equity ratio of 2. The debt is almost risk-free. Estimate the beta for your equity if projects have constant betas, but your firm will carry a debt-equity ratio of 1/2. (Hint: To translate a debt-equity ratio into a debt-asset ratio, make up an example.)

Q 10.36. A Fortune 100 firm is financed with $15 billion in debt and $5 billion in equity. Its historical equity beta has been 2. If the firm were to increase its leverage from $15 billion to $18 billion and use the cash to repurchase shares, what would you expect its levered equity beta to be?

Q 10.37. The prevailing risk-free rate is 5% per annum. A competitor to your own firm, though publicly traded, has been using an overall project cost of capital of 12% per annum. The competitor is financed by $1/3$ debt and $2/3$ equity. This firm has had an estimated equity beta of 1.5. What is it using as its equity premium estimate?

Q 10.38. Draw some possible security markets lines (SML's) that would not be consistent with the CAPM. On the x axis, put the true market beta. On the y axis, put the true expected rate of return.

Q 10.39. Under what circumstances is the CAPM a good model to use? What are the main arguments in favor of using it? When is it not a good model?

Q 10.40. If you use the CAPM, explain for what kinds of projects it is important to get accurate equity-premium estimates.

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