Basic Notation, Returns, and Compounding

(Welch, Chapter 02)

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Motivating Questions

- Can you add rates of return and interest rates?
- Can you average interest rates?
- If the bank posts an 8% interest rate, how much money will \$100 give you at year-end?
- Do fast-growing companies earn more return then slow-growing ones?

A Perfect Market for Capital

For the next few chapters, we pretend we live in a *perfect market* for the provision of investment capital (money).

A *perfect capital market* (**PCM**) must satisfy four assumptions.

PCM = Perfect Capital Market

- (not a common abbreviation; just us)

PCM Assumptions I

1. **No differences in opinion**
   - Uncertainty is okay, but everyone must agree to exactly what it is. We must not have different information or opinions

PCM Assumptions II

2. **No taxes**
   - or government interference or regulation [except government enforces property rights]

PCM Assumptions III

3. **No transactions costs**
   - Neither direct nor indirect

PCM Assumptions IV

4. No big sellers/buyers

- There must always be more where they came from. No (few) investors or firms are special. If investors differ, there must be infinitely many clones competing for each type of investor.

Why Assume a PCM?

Any analysis is easier in a PCM, so start with it.

Any logic that fails even in a PCM will surely fail (be wrong) in a more realistic market.

- Put differently, as real-world financial markets become closer to perfection (and some do), any methods we would use would need to converge to the PCM solution,
- . . . until they converge perfectly in the limit.

Why Not an ImPCM?

ICM = Imperfect Capital Market

- (not a common abbreviation; just us)

Ch. 12 & 13 will explain why.

- PCM makes borrowing and lending rates equal.
- Methods become more difficult and non-general
- **Preview**: Value/prices can depend on the owner. . . Yikes! No more objective price!

Some markets are indeed less perfect than others.

Realism varies.

Real vs Nominal Rates

Unless otherwise specified as "real", all quantities are nominal.

- This is not just for class but standard convention.
- If the interest rate is 15%, it means 15% nominal.
- i.e., we quote returns in terms of currency units, not in terms of apples.

We will discuss inflation in Chapter 4.

- **Preview**: presumably, nominal rates are set partly by the expectation of future inflation.

Extra Chapter 2 Assumption

In Chapter 2, we assume **perfect certainty**.

- We *know* what the rate of return (**RoR**) on every investment project will be.
- No need to worry about statistics
- No need to worry about investor risk preferences.
- All same-period (interest) RoRs must be the same.

This assumption is only to start the exposition.

Extra Extra Chapter 2 Assumption

In Chapter 2, we assume *equal* RoRs.

- . . . per period, of course!

Examples:

- A 1-year bond offers 10%,
- a 1-year bond next year offers 10%,
- a 2-year bond offers 21%.
- a 1-year bond in 10 years offers 10%.
- a 30-year bond offers 10% *per year*.

Eliminates concern for the "yield curve" (Ch. 5)

Notation I (Time)

Time Convention:

- 0 = today, right now
- 1 = next period (e.g., day, year, etc.)
- t = some time period (in the future)
- T = often to denote a final time period

Notation II (Flow vs Stock)

Flow: something accumulating over a time span.

Stock: an "instant moment" snapshot quantity.

Examples:

- Firm assets are a stock. Earnings are a flow.
- A *price* is a stock. A *rate of return* (*RoR*) is a flow.

The distinction is not always so clear.

- **Example**: Dividends
- if they accrue, use two subscripts;
- if its instance of payment, use only one subscript.

Notation III (Variable Names)

- C = cash amount
- CF = cash flow (last instant?)
- C_t = instant cash amount at time t (or at end)
- $D_{t-1,t}$ = a flow of D (e.g., dividends) from t-1 to t
- D_t = common notation for $D_{t-1,t}$
- $D_{15,20}$ = a flow of D from t=15 to t=20
- Return vs Net Return vs. Rate of Return
- r, r_1, $r_{15,20}$, r_4: all RoRs.
- RoR abbreviates *rate of return*.

Jargon Footnotes

If an investment is a loan, the rate of return (RoR) is usually called an interest rate.

- either "RoR" or "interest rate" are correct.
- Verbal statements are often unclear:
  - a return (CF_1),
  - a net return ($CF_1 - CF_0$),
  - and a (net) RoR ($CF_1/CF_0 - 1$),
  - you are usually assumed to know what the speaker means.

Rate of Return (RoR) Formula

The RoR from investing CF_0 today and getting CF_1 at time 1 is

$$r = r_{0,1} = \frac{(CF_1 - CF_0)}{CF_0} = \frac{CF_1}{CF_0} - 1 \ .$$

This could be called

The Fundamental Formula of Finance

This rate of return (RoR) is also called a "*holding* rate of return."

RoR Formula Rewrite

$$r_{0,1} = \frac{(CF_1 + D_{0,1} - CF_0)}{CF_0} = \frac{(CF_1 + D_{0,1})}{CF_0} - 1 \ .$$

- it assumes no interim reinvestment of dividends D (or coupons or rent)—as if dividends were paid at the *end* of the period.

Using our convenient abbreviations,

$$r_1 = \frac{(CF_1 + D_1 - CF_0)}{CF_0} = \frac{(CF_1 + D_1)}{CF_0} - 1 \ .$$

RoR Components

- *Dividend yield*: $D_{0,1}/CF_0$
  - For bonds, this is called the *coupon yield*.
- *Capital gain*: CF_1 - CF_0
- *Percent price change*: $(CF_1 - CF_0)/CF_0$
- *(Total) RoR*: percent price change plus the interim payment yield.

Warning:

- $r_1 = P_1/P_0 - 1$ is often assumed to include all interim payments in P_1.

Percent Price Changes

If the RoR is positive, can the percent price change be negative?

Percent Price Changes

If the RoR is negative, can the percent price change be positive?

Holding RoR

If you invest \$5 and will receive 8 in 10 years, what is your (holding) RoR?

Negative Rates of Return?

Can a RoR be negative?

Negative Interest Rates?

Can an *interest* rate be negative?

Negative *Ex-Ante* Interest Rates?

Can interest rates be negative *ex-ante?*

- recall: we mean nominal unless otherwise specified.

Application: Today's Rf?

What is today's prevailing interest rate?

Compare 10% to 5%

Would you say that

- 10% is 5% more than 5%, or that
- 10% is 100% more than 5%?

Points and Basic Points ("bp")

- Rate changes can be easily misunderstood, which is why **points** and **basis points** (bp) were invented:
- 1 full **point** means 1%.
- 1 **basis point** ("bip") means 0.01%.
- Examples:
  - The difference between 5% and 10% is 5 points.
  - The difference between 5% and 10% is 500 bp.

Practice

If you invest $55,000 at an interest rate of 350 basis points above the 5% interest rate, what will you receive at the end of the period?

(This is called the future value of money, FVM.)

Practice

If you have \$5 and you earn a RoR of 250%, how much money will you have?

Practice

If you have \$5 and you earn a RoR of 40%, how much money will you have?

Future Value

What is the formula for the FV (Future Value) of money? How does it relate to the RoR formula?

FV Example

If you have \$5 and you earn a RoR of 20% in the first year and a RoR of 20% the following year, how much money will you have?

FV Example

If you have \$5 and you earn a RoR of 20% in each year, how much money will you have in x years?

Graph: Compounding at 20%

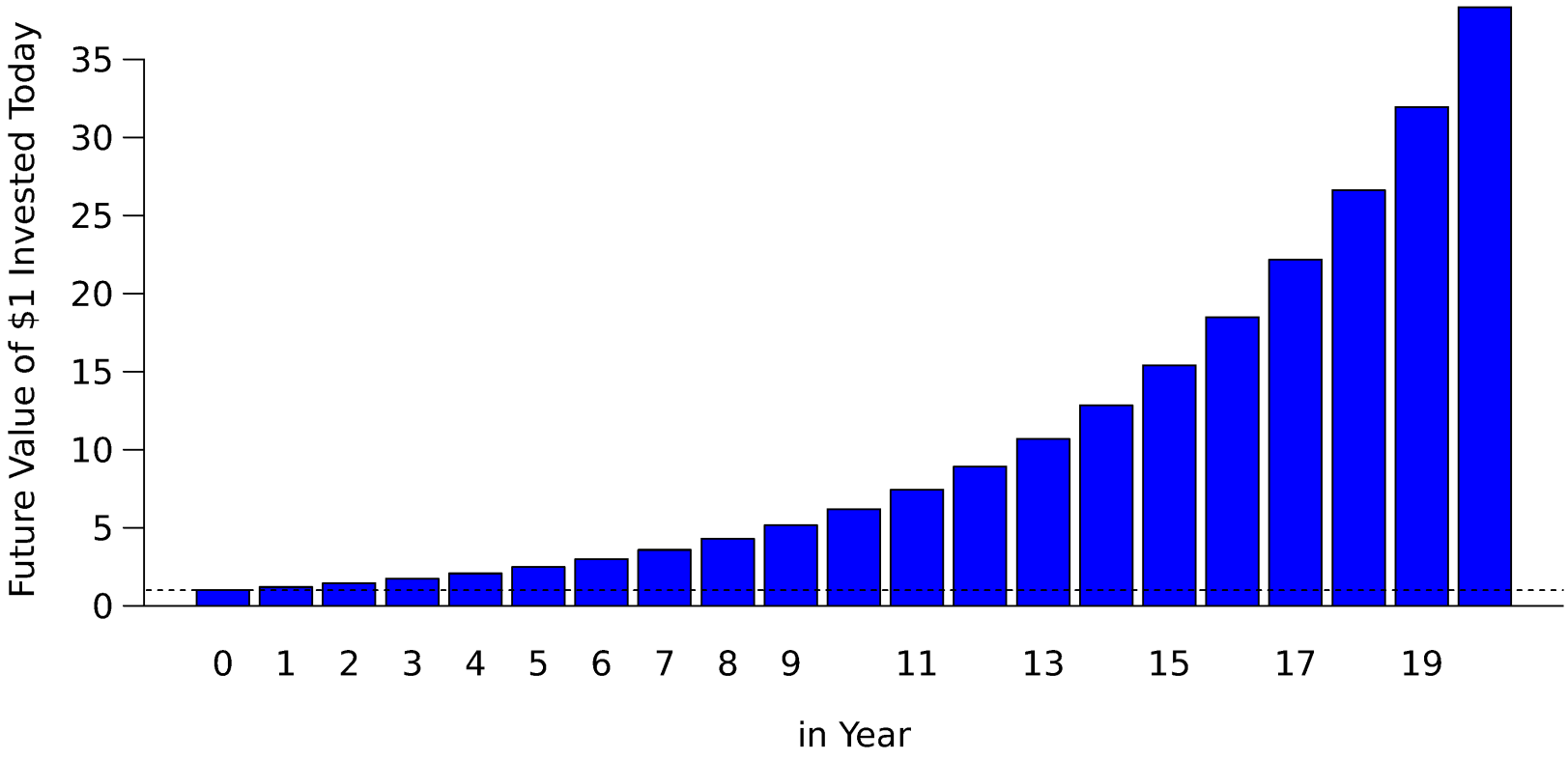

Figure 1: compounding returns

Preview: Different Interest Rates

If the 1-year interest rate is 20% this year, how much money will you get for a 500 investment today in one year?

If the following 1-year interest will be 50%, how much money will you have in/after 2 years?

Sequential RoRs

What is the total holding RoR from one RoR of 20% followed by one of of 50%?

Is it $50\% + 20\% = 70\%$?

Sequential RoR Formula

What is the formula for the total holding RoR, given the two individual RoRs?

The Compounding Formula

The Compounding Formula:

$$r_{0,x} = (1 + r_{0,1}) \cdot (1 + r_{1,2}) \ldots \cdot (1 + r_{x-1,x}) \ .$$

If the interest rate remains constant,
$r_{t,t+1} = r_{0,1} = r$ for all T, then

$$r_{0,T} = (1 + r)^T \ .$$

WRONG FORMULA!

The compounding formula is

$$1 + r_{0,x} = (1 + r_{0,1}) \cdot (1 + r_{1,2}) \ldots \cdot (1 + r_{x-1,x}) \ .$$

If the interest rate remains constant,
$r_t, t+1 = r_{0,1} = r$ for all T, then

$$1 + r_{0,T} = (1 + r)^T \ .$$

When I introduce errors, you must catch them!