Uncertainty, Default, and Risk

(Welch, Chapter 06-A)

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Maintained Assumptions

Perfect Markets

1. No differences in opinion.
2. No taxes.
3. No transaction costs.
4. No big sellers/buyers—infinitely many clones that can buy or sell.

BUT NO LONGER Certainty

Sadistics

We now need to predict (describe) the future.

For this, we need statistics.

Legal Discliamer

This chapter may be illegal in some states. Attend the rest of this course at your own risk.

For example:

> *Persons pretending to forecast the future shall be considered disorderly under subdivision 3, section 901 of the criminal code and liable to a fine of $ 250 and/or six months in prison.*
> *(Section 889, New York State Code of Criminal Procedure.)*

Statistics

Covered more fully in your sadistics course.
This brisk overview is just the minimum needed.

Random Variables (RVs)

A **Random Variable (RV)** (often with tilde over the variable) is

- not an ordinary algebra variable,
  - that has some specific unknown value,
- but best thought-of as a histogram,
  - a cloud function of a randomizing device (e.g., a coin), which can give it some specific value with histogram probabilities soon.

Inference

In many applications, statisticians *assume* you understand the system and know the histogram, but not the draw (the outcome).

- **Great for**: coin, die, roulette.
- **Sucks for**: stock market (where we do not understand the underlying physics).
- On occasion, I will warn you.

Die Throw RV

Our main random variable example will be the payoff you get after a die is thrown:

- $1 = -\$6$,
- $2 = +\$36$,
- $3 = -\$12$,
- 4 to $6 = +\$150$.

Let's call this variable D (ie).

Draw its histogram.

The Histogram of D

- Draw Please

The Expected Value

$E(\cdot)$ is common notation for *Expected Value*.

- Example: $E(D)$ is the expected value of D.

Think of the expected value as **the mean of the RV if infinitely repeatedly drawn**.

- *average*, or *mean*, or *expected* are often used interchangeably, even if not fully correct.

You can calculate the EV from the histogram

- multiply each outcome by its probability,
- and add up the terms.

Calculate E(RV)

What is the expected payoff of D?

Question

Is $E(D)$ a number or a RV?

Mean Mean

Is the expected payoff always the most likely outcome?

Are half of all outcomes always below the mean?

Central Statistics

What is the mean number of testicles per human?
What is the median?
What is the mode?
(What is the standard deviation?)

Functions of RVs

The $E(D)$ was \$78.

What is the expected value of the die-squared, $E(D^2)$?

Recall D

- $1 =$ –\$6,
- $2 =$ \$36,
- $3 =$ –\$12,
- 4 to $6 =$ \$150.

Linear Functions of RVs

Is $E[D^2]$ the same as $[E(D)]^2$?

PS: Is $2 \times E(D) + 1 = E(2 \times D + 1)$

Measuring Unexpected?

What is an unexpected quantity?

Fair Bets

What is a fair bet?

What would it take for the above die-throw to become a fair bet?

Variance

The variance is $Var(D) = E\{[D - E(D)]^2\}$

Roughly speaking, the variance is the **expected squared deviation from the mean**.

- This is just pseudo-intuitive.
- The units on and meanings of variances are usually incomprehensible.
- Infinitely many draws: *Mean* of $(D - Mean(D))^2$.

Variance Example

From the RV's histogram, to obtain the variance, multiply each squared deviation from the mean by its probability, and add up terms.

State	Prob	Outcome	Outcome-Mean	Square
"1"	1/6	–$6	–$84	
"2"	1/6	$36	–$42	
"3"	1/6	–$12	–$90	
"4"-"6"	3/6	$150	+$73	
Weighted Mean:		$78	$0	

Standard Deviation (SD)

The square-root of the variance:

$$\text{SD} = \sqrt{\$\$5,412} \approx \$73.57 \ .$$

Think of the SD as the **typical deviation from the mean** of the next draw.

- Not entirely correct, but close enough.
- SDs have meaningful units.

Variance or SD?

If the variance is higher, then the standard deviation is higher and vice-versa.

- So, risk is either VAR or SD.
- SD is more meaningful,
- but VAR is also commonly used.
- Vanilla or Chocolate?

Nerd: Signal Balance

Let's say you want to combined two signals equally

It is important that you normalize both of them first. Otherwise, you may give too much weight to the signal that has the higher mean—or, if means are the same, that has the higher variance.

Two common normalizations:

1. the "percentile rank"
2. the "normalized value" (net of mean, divided by standard deviation).

Big Leap of Faith

What is the True Histogram?

- Yes, we know the histogram for a die throw from physics.
- But what is the histogram for the RoR on the stock market?
- Therefore, we pretend that the (many) historical outcome realizations of RoRs can proxy for the true unknown histogram of RoRs.

Historical = Future?

- Then we pretend that this historical histogram applies to future RoRs, too.
- This is a "Hail Mary."
  - But once we assume we know the histogram, we can calculate expectations and do much more!
  - Employment program for finance professors?

Future Stock Returns as RVs

This translation of the historical outcome histogram (distribution) into the future outcome histogram (distribution) is a *heroic* assumption.

- But what else can we do?
- We have no better ideas.
- Always remain mindful of the problem!

Expected Stock Returns

Extrapolation works poorly for mean returns.

- Investors try to learn and adapt all the time.
- Survivorship Bias.
- If AAPL was a most profitable investment in the past, will it be in the future?
- If IBM was great, will it remain great?

Expected Market Returns

What is the expected RoR on the overall stock market (say, the S&P500 or DJ30)?

Is it still the same that it was historically?

Why?

Risk of Stock Returns

Extrapolation works well for variances, standard deviations, and market-betas (TBD).

- Not perfect, but well.
- Statistics works well for risk management!

Historical Example

What are the $E(R)$ and $SD(R)$ if you only knew four historical stock returns:

- $+10\%$,
- -5%,
- $+20\%$,
- $+15\%$,

Sample Historical Calculations

Mean: 10%
Each Deviation: 0%, 15%, 10%, 5%
Squared Deviations: 0, 225, 100, and 25,
Sum Squared Deviations: 350.

- Now take the mean?!

N vs N-1

If these were true population probs, then divide this sum by N=4, which gives you 87.5.

- This translates into an sd of 9.35%.

If these were just historical samples, then divide sum by N-1=3, which gives you 116.67.

- This translates into an sd of 10.8%.

Excel

Excel *stdev()* uses N-1.

- Excel assumes you knew only the sample.

This makes little difference for large datasets.

- But, it matters with $textN = 4$.

(N vs $N-1$ matters only for small samples.)

Excel: *stdevp()* is not *stdev()*;

Excel: *varp()* is not *var()*.

Repeat Warning

Do not trust the historical means blindly for predicting future expected RoRs.

For individual stocks (rather than big diversified portfolios), this would be exceedingly stupid.

Even for big diversified portfolios, this is a big leap of faith.

BUT NOT ALL IS LOST

(Recent) historical variances (and covariances and standard deviations) are usually good predictors of (short-term) future variances (and covariances and standard deviations).

- Advice: use 1–3 years of historical *daily* return data to estimate them.

Estimating Tail Risk?

Unfortunately, a short time-series of historical numbers is usually not reliable enough to calculate tail-risk—the probability of a complete blow-up.

- How do you estimate the risk of the next Space-X rocket exploding?
- How do you estimate the prob of the next meteor hitting NYC from 80 years of data?
- We would need more data than we have!
- Alternative: Options.

Context Switch

Investor Preferences

Preferences over Bets

If I offer you a bet of $+\$1$ if heads and $-\$1$ if tails, you pick a coin and someone else in class to throw it (at least 5 yards!), would you be willing to take this bet?

If not, how much would I have to pay you?

Low Risk-Aversion

When is risk neutrality (or low risk-aversion) a good assumption?

Risk

Why do people climb mountains, play the lottery, drive motorcyles, or fly airplanes?

Risk Preferences

- **Most of finance assumes that investors are risk-averse.**
- **When choosing between investments with equal expected returns, investors prefer the lowest-risk one(s).**
- **Risk can be measured by portfolio SD.**

Enough Theory

now come applications of these concepts. . . !