Investing Choices and Risk Measures

(Welch, Chapter 08)

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Maintained Assumptions

Perfect Markets

1. No differences in opinion.
2. No taxes.
3. No transaction costs.
4. No big sellers/buyers—infinitely many clones that can buy or sell.

With risk and risk aversion

Investors

How should investors choose among many different projects?

Corporate Managers

How do projects determine company risk?

How do investors think?

What is your opportunity cost of capital $E(r)$?

Risk Characterization

We use the SD of portfolio return.

Investments

Four equally likely scenarios:

- states: yellow, red, green, blue.
- "state-based" preferences are more general than our Mean/SD preferences, but more general.

Four investment assets: A, B, C, D.

Returns (in Percent or Dollars).

Investment Contingencies

```
   Ylw    Red    Grn    Blu
-----------------------------
A: -4.0   -4.0   +6.0   +6.0
B: -1.0   +9.0   +9.0   -1.0
C: -1.25  +1.25  +3.75  +1.25
D: +3.0  +13.0   +3.0   -7.0
-----------------------------
```

Investment Rewards

What are the rewards of the four investments?

Investment Risks

What are the risks of the four investments?

Population vs Sample Statistics

If Ylw-Blu returns are just representative historical realizations, you would divide by 3, not 4 in your computation of the variance.

In real life, we rarely have population statistics.

- Historical are sadly our best choice.

Overall vs Parts Risk

The standard deviation is a meaningful measure of risk only for your overall portfolio.

You should not care about the standard deviations of your individual investments.

Means and SDs of 4 Assets

```
   Mean   x-Mean  Var     SD
-----------------------------
A:
B:
C:
D:
-----------------------------
```

Portfolio Risk

What is the risk of an (equal-weighted) portfolio of Asset A and Asset B?

- (HINT: First compute the RoRs of the combination portfolio in each state.)

Portfolio Risk

Is the average portfolio or are the individual components riskier?

Why?

Good Portfolios?

What kind of portfolio would you—a smart but risk-averse investor—hold?

Real Life Prime Portfolio

In real life, what portfolios should and do smart investors with risk-aversion hold?

Portfolio Risk

What is A's portfolio risk if you add C to your portfolio vs if you add D to your portfolio?

```
       Mean   x-Mean  Var    SD
------------------------------
A+C:
A+D:
------------------------------
```

Riskier Investment I

Is C or D the riskier investment in itself?

Riskier Investment II

If you already own A, is C or D the riskier addition?

Why?

Base Portfolio

If investors are smart, what is their base portfolio A?

CAPCM Preview

Advance Guess: If you are selling to smart investors either C or D, for which of these two projects do you think will investors clamor to invest in your project (i.e., accept a lower expected RoR)?

Fundamental Investment Insight

Investors (should) care about overall portfolio risk, not about the constituent component risk.

From a corporate managerial perspective, it is not low-risk projects that investors like, **BUT** projects which wiggle opposite to the rest of their portfolios.

Synchronicity

How should we measure synchronicity?

- For exposition, consider A to be the market portfolio that investors are already holding.
- We need a measure of how synchronous or non-synchronous any new stock/asset/project is with respect to this portfolio A=M.

Calculate Ri-Mean(Ri)

```
   Ylw    Red    Grn    Blu
-----------------------------
A: -5.0   -5.0   +5.0   +5.0
B: -5.0   +5.0   +5.0   -5.0
C: -2.5    0.0   +2.5   +0.0
D:  0.0  +10.0   +0.0  -10.0
-----------------------------
```

Calc COVAR, CORR, BETA

Covariance is mean of cross-products:

1. $$(A-\overline{A})\times(B-\overline{B}) = +25, -25, +25, -25.$$

2. $$(A-\overline{A})\times(C-\overline{C}) = +12.5, 0, +12.5, 0.$$

3. $$(A-\overline{A})\times(D-\overline{D}) = +0, -50, 0, -50.$$

cov(A,B)=0, cov(A,C)= +6.25, cov(A,D)= –25.

- why are we demeaning and multiplying?

Beta (Slope)

Beta is the covariance divided by the variance:

$$\beta_{C,A} = 6.25/25 = 0.25 \; ,$$

$$C = 1 + 0.25 \cdot A, \qquad (A = -1.5 + 2 \cdot C) \; .$$

$$\beta_{D,A} = -25/25 = -1,$$

$$D = 4 - 1 \cdot A, \qquad (A = 2.5 - 0.5 \cdot D) \; .$$

Correlation

The correlation is the covariance divided by the standard deviations of its two ingredients:

$$cor(A, C) \approx 0.7071 \qquad cor(A, D) \approx -0.7071 \; .$$

- The order does not matter for covariance or correlation. It matters only for beta.

Risk Contribution Measures?

Covariance generalizes variance. (Why?) Thus, it also has uninterpretable units. Yuck.
Correlation has a scale problem.

- A 1 cent investment has the same correlation as \$1 million investment.
- But the 1-cent would contribute less risk!

Best Measure: Market-Beta

The best risk contribution of adding B to M is B's market-beta with respect to M.

- This means $var(Rm)$ is the denominator.
- Without verbal qualification, beta always means with respect to Rm, i.e., market-beta.

Happy Family

Covariance, correlation, and beta always have the same sign.

They differ by magnitude.

Beta is Slope

Beta is a slope. Put A (M) on the X axis, and your project B (or C) on the Y axis.

- A slope of 1 is a diagonal line.
- A slope of 0 is a horizontal line.
- A slope of ∞ is a vertical line.

Without alpha, beta tells you how an x% higher RoR (than normal) in the market will likely reflect itself simultaneously in a $\beta_i \cdot x\%$ higher rate of return (than normal) in your stock.

Beta Interpretation

The stock-return beta helps with a conditional forecast of Ri, given Rm.

Mediocre measures of market-beta are available on every finance website.

A better measure would use daily stock returns on 1-2 years of historical data.

The best market-beta measure winsorizes smartly.

- winsorizing means trimming to limits.

Market-Beta of Market

What is the market-beta of the overall stock market (say, the S&P500)?

Market-Beta of Risk-Free Rate

What is the market-beta of the risk-free rate?

High vs Low *Beta* Projects

Given equal expected returns, what's more desirable?

- A project with a high beta? Or
- A project with a low beta?

High vs Low *Risk* Projects

Should high or low variance projects have to offer higher average RoRs?

High vs Low *Beta* Projects

Should high or low beta projects have to offer higher average RoRs?

Conglomeration

New Firm: 40% C and 60% D. ($4m and $6m.)
What is the average RoR (mean)?

What is the average variance?

What is the average sd?

What is the average beta?

Value-Averaging

Which statistics can you "value-average"?
Which statistics can you not "value-average."

Corporate Market Beta

Is there a quicker way to compute the overall market-beta of your firm, based on the market-betas of its constituent projects?

Warning: Time-Changing Pfio Weights

Portfolios and firms have changing investment weights every instant.

This means that you cannot use today's investment weight retroactively.

Mean-Variance Frontier

(Omitted.)

The mean-variance efficient frontier ($=$ the mean-standard deviation efficient frontier).

- optimal combination of assets.
- covering it would require 2+ full lectures.
- Underlies CAPCM+. Take an investments course!
- It is in common (practical) use.
- Important.

Variance of Weighted Sum

If pfio P consists of two assets:

$$r_P = w_A \cdot r_A + w_B \cdot r_B \ ,$$

the formula for the portfolio variance is

$$Var(r_P) = Var(w_A \cdot r_A + w_B \cdot r_B) =$$

$$w_A^2 \cdot Var(r_A) + w_B^2 \cdot Var(r_B) +$$

$$2 \cdot w_A \cdot w_B \cdot Cov(r_A, r_B) \ .$$

- This is *not* $w_A \cdot Var(R_A) + w_B \cdot Var(R_B)$!
- You cannot value-weight variances!

Effect of Changing Weights

The generalized formula is based on variance-covariance matrix between all assets and your investment weights.

It makes it easy to recompute the portfolio risk when you change portfolio weights.

Time Correlation

What is the correlation of stocks' RoRs from one day to the next day?

Time-Adjusting Risk

If the risk of investing in x for 1 year is $\sigma = 20\%$, what is the risk of investing for 10 years?

(This is an important application.)

A1: Constant Risk

Let's assume that the per-unit-of-time standard deviation remains constant.

- Omit time subscript.
- Let's just call this number σ.

A2: Uncorrelated over Time I

Rates of return over time should be uncorr.

- If not, non-zero is likely statistical noise.
- Otherwise, you could use past stock returns to outpredict future stock returns.

Algebraically,

$$Cov(R_t, R_{t+s}) \approx 0 \ ,$$

where the subscripts t and $t+s$ refer to two time periods, not to different stocks.

A2: Uncorrelated over Time II

In this case, the following approximation is not bad:

$$\boxed{Sdv(R_{0,T}) \approx \sqrt{T} \cdot \sigma}$$

Example: if your portfolio risk is 10% per month, then your annual risk is about $\sqrt{12} \cdot 10\% \approx 35\%$ per year.

Time-Adjusted Derivation

$$Var(R_{0,T}) \approx Var(R_{0,1} + R_{1,2} + ... + R_{T-1,T})$$

$$= Var(R_{0,1}) + Var(R_{1,2}) + ... + Var(R_{T-1,T})$$

$$= \text{many 0 covariance terms}$$

$$\approx T \cdot \sigma \ .$$

Sharpe Ratio

Sharpe-Ratio (SR): a (badly flawed but common) measure of investment performance:

$$\text{SR}_i = \frac{\overline{R}_i - Rf}{SD(R_i)} = \frac{\overline{R_i - Rf}}{SD(R_i - Rf)} \ .$$

- The SR grows with the square-root of time.
- Calculated typically from *monthly* RoRs annualized by $\sqrt{12}$.
- Historical SR of Market: $4\%/10\% \approx 0.4$.

Nerd: VW/EW Portfolio Maintenance

Is it easier to maintain a value-weighted or an equal-weighted portfolio?