Capital Budgeting: Disribution Moments

(Welch, Chapter 13-1)

Ivo Welch

Expected Cash Flows

Value of Graduate Degree

Life is not fair. At age 25,

- male life expectancy is about 52.5 years,
- female life expectancy is about 57 years.

If you expect to earn \$100,000 per year **extra** because of your degree, and

if the applicable interest rate / (opportunity) cost of capital is 5%/year,

then what is your degree really worth?

Expectation of Function

Is $E(f[x]) = f[E(x)]$?

Where could this even matter?

How would you figure out the answer for yourself?

Expectation Cooking

Cook up as simple an example as possible!

Say, you can expect to live one of three lengths, with equal probabilities of 0, 1, and 2 years.

Say, the interest rate is 10%.

What is the correct expected value of \$1 as long as you live?

Simple Stew I

$$E(CF(t)) = 1/3 \times (\$1 + \$1/1.1 + \$1/1.1^2)$$

$$+1/3 \times (\$1 + \$1/1.1) + 1/3 \times (\$1)$$

$$= \$1.88 \ .$$

However, if you lived the expected number of years, you would receive

$$CF(E(t)) = \$1 + \$1/1.1 = \$1.91 \ .$$

\$1.88 and \$1.91 are *not* the same!

Simple Stew II

Non-linearities in E(CF) matter greatly in insurance, annuities, etc.

But non-linearities also matter when machines can wear out.

Sometimes they are first-order important. Sometimes approximations are reasonable. Tough to know beforehand!

Drug-Dealing

You expect to earn \$300,000/year. Holding money for a drug-dealer would earn you an extra \$100,000 per year. It is highly unlikely that you will get caught—maybe 1 in 1,000. 999 out of 1,000 times, you earn 25% more.

- $.999 \times \$400 + .001 \times \$0 \approx \$400$.

Bad Example

OK, drug-dealing was a bad example but it illustrated the possibility of a rare bad outcomes with terrible payoffs.

More commonly, there is a small probability of disaster where you lose all. The manager dies. An explosion happens. etc.

It is rare that these are positive surprises.